PROPOSED
SYLLABUS FOR
Master of Science

MATHEMATICS

Under CBCS guidelines

2023

COURSE STRUCTURE

SEMESTER	COURSE CODE	COURSE TITLE	CREDITS
I	MMAC 1.11	Ordinary Differential Equations (Theory)	4
	MMAC 1.12	Ordinary Differential Equations (Practical)	2
	MMAC 1.21	Linear Algebra	5+1
	MMAC 1.31	Real Analysis	5+1
	MMAC 1.41	Abstract Algebra	5+1
II	MMAC2.11	Numerical Analysis	4
	MMAC 2.12	Programming in C (Practical)	2
	MMAC2.21	General Topology	5+1
	MMAC2.31	Classical Mechanics	5+1
	MMAC2.41	Complex Analysis	5+1
III	MMAC 3.11	Partial Differential Equations (Theory)	4
	MMAC 3.12	Partial Differential Equations (Practical)	2
	MMAC 3.21	Functional Analysis	5+1
	MMAD 3.11	OPTIONAL*	5+1
	MMAD 3.21	OPTIONAL*	5+1
IV	MMAC 4.11	Mathematical Methods (Theory)	4
	MMAC 4.12	Mathematical Methods (Practical)	2
	MMAC 4.21	Rings & Modules	5+1
	MMAD 4.11	OPTIONAL**	5+1
	MMAD 4.21	OPTIONAL**	5+1 /6

*** DISCIPLINE SPECIFIC ELECTIVE 1& 2**

MMAD 3.11 & MMAD 3.21

COURSE TITLE	CREDITS
Number Theory	5+1
Operation Research	5+1
Tensor Analysis & Riemannian Geometry	5+1
Measure Theory	5+1
Graph Theory	5+1
Mathematical Statistics	5+1
Field Theory	5+1
Mathematical Modeling	5+1
Multivariable Calculus	5+1

**** DISCIPLINE SPECIFIC ELECTIVE 3 &4**

MMAD 4.11 & MMAD 4.21

COURSE TITLE	CREDITS
Fluid Mechanics	5+1
Fourier Analysis	5+1
Algebraic Number Theory	5+1
Analytic Number Theory	5+1
Algebraic Topology	5+1
Differential Geometry of Manifolds	5+1
Commutative Algebra	5+1
Discrete Mathematics	5+1
Lie Algebra	5+1
Theory of Relativity	5+1
Game Theory	5+1
Dissertation/Project	6

SEMESTER – I

MMAC 1.11
ORDINARY DIFFERENTIAL EQUATIONS

Theory Credit: 4

UNIT I Linear equations with constant coefficients; the second and higher order homogeneous equation; initial value problems for second order equations; existence theorem; uniqueness theorem; linear dependence and independence of solutions; the Wronskian and linear independence; a formula for the Wronskian; the non-homogeneous equation of order two.

UNIT II Linear equations with variable coefficients, initial value problems for the homogeneous equations; existence theorem; uniqueness theorem; solutions of homogeneous equations; the theorem on n linearly independent solutions; the Wronskian and linear independence;

UNIT III Non-homogeneous equations; homogeneous equations with analytic coefficients; Legendre equation, justification of power series method; Legendre polynomials and Rodrigues' formulae.

UNIT IV Existence and uniqueness of solutions – introduction; equations with variable separated; exact equations, Lipschitz condition; non-local existence of solutions; uniqueness of solutions; existence and uniqueness theorem for first order equations; statement of existence and uniqueness theorem for the solutions of ordinary differential equation of order n.

UNIT V Linear equations with regular singular points – introduction; Euler equation; second order equations with regular singular points – example and the general case, convergence proof, exceptional cases; Bessel equation; regular singular points at infinity.

Recommended Books and References:

1. E. A. Coddington - An Introduction to Ordinary Differential Equations, 2001, Prentice-Hall of India Private Ltd., New Delhi, ISBN-10:0486659429
2. W. T. Martainand E. Relssner, Elementary Differential Equations., 1995 (3rd Edition), Addison Wesley Publishing Company, Inc, ISBN-9780201045031
3. E. A. Codington and N. Levinson, Theory of Ordinary Differential Equations, 1999, Tata McGraw hill Publishing co. Ltd. New Delhi, ISBN-9780070992566
4. G F Simmons, Differential equation with applications and historical Notes, 2017, 2nd Ed, McGraw Hill Education, ISBN-9780070530713

MMAC 1.21
LINEAR ALGEBRA

Theory Credit: 5
Tutorial Credit: 1

UNIT I Vector spaces, linear independence; linear transformations, matrix representation of a linear transformation; isomorphism between the algebra of linear transformations and that of matrices;

UNIT II Similarity of matrices and linear transformations; trace of matrices and linear transformations, characteristic roots and characteristic vectors, characteristic polynomials, relation between characteristic polynomial and minimal polynomial; Cayley-Hamilton theorem (statement and illustrations only); diagonalizability, necessary and sufficient condition for diagonalizability;

UNIT III Projections and their relation with direct sum decomposition of vector spaces; invariant subspaces; primary decomposition theorem, cyclic subspaces; companion matrices; a proof of Cayley-Hamilton theorem; triangulability; canonical forms of nilpotent transformations; Jordan canonical forms; rational canonical forms.

UNIT IV Inner product spaces, properties of inner products and norms, Cauchy-Schwarz inequality; orthogonality and orthogonal complements, orthonormal basis, Gram-Schmidt process; adjoint of a linear transformation; Hermitian, unitary and normal transformations and their diagonalizations.

UNIT V Forms on inner product spaces and their matrix representations; bilinear forms; Hermitian forms; symmetric bilinear forms; orthogonal diagonalization of real quadratic forms.

Recommended Books and References:

1. K. Hoffman and R. Kunze, Linear Algebra (2nd edition), Prentice Hall of India Pvt. Ltd., New Delhi, 2000.
2. S. Axler, Linear Algebra Done Right 3rd Edition
3. P. B. Bhattacharya, S. K. Jain and S. R. Nagpal,First Course in Linear Algebra, Wiley Eastern Ltd., New Delhi, 2000.
4. I. N. Herstein, Topics in Algebra (4th edition), Wiley Eastern Limited, NewDelhi, 2003.
5. G. E. Shilov,Linear Algebra, Prentice Hall, 1998.
6. P. R. Halmos, Finite Dimensional Vector Spaces, Va n Nostrand Inc., 1965.
7. D. T. Finkbeiner, D.B. Taraporevala, Introduction to Matrices and Linear Transformations (3rd edition), Bombay, 1990.
8. S. Kumaresan, Linear Algebra, A Geometric Approach, Prentice-Hall of India Pvt. Ltd., New Delhi, 2001.

MMAC 1.31
REAL ANALYSIS

Theory Credit: 5
Tutorial Credit: 1

UNIT I The real and complex number systems: Elements of set theory, ordered sets, fields, the real field, the complex field, Euclidean spaces, Basic topology: Finite, countable and uncountable sets, metric spaces, compact sets, perfect sets, connected sets.

UNIT II Numerical sequences and series:Convergent sequences, subsequences, Cauchy sequences, upper and lower limits, series, series of non-negative terms, the number *e*, the root and ratio tests, power series, summation by parts, absolute convergence, addition and multiplication of series, rearrangements.

UNIT III Limits of functions: continuous functions, continuity and compactness, continuity and connectedness, discontinuities, monotonic functions, infinite limits and limits at infinity.
The derivative of a real function, mean value theorems, the continuity of derivative, l'Hospital's rule, derivatives of higher order, Taylor's theorem, differentiation of vector-valued functions.

UNIT IV The Riemann-Stieltjes integral: Definition and existence of the integral, properties of the integral, integration and differentiation, integration of vector-valued functions, rectifiable curves.

UNIT V Sequences and series of functions:Uniform convergence, uniform convergence and continuity, uniform convergence and integration, uniform convergence and differentiation, equicontinuous families of functions, the Stone-Weierstrass theorem.

Recommended Books and References:

1. Walter Rudin, *Principles of Mathematical Analysis*, 3rd Edition, McGraw Hill Education, 1976, ISBN-9781259064784
2. Robert G. Bartle, *The Elements of Real Analysis*, 2nd Edition, John Wiley & Sons, 1975, ISBN-9780471054641
3. Tom M. Apostol, *Mathematical Analysis*, 2nd Edition, Addison-Wesley Publishing Company, Inc., 1974, ISBN-9788185015668
4. Terence Tao, *Analysis I*, 3rd edition, Hindustan Book Agency, 2014, ISBN-9789380250649
5. Terence Tao, *Analysis II*, 3rd edition, Hindustan Book Agency, 2014, ISBN-9789380250656
6. S Kumaresan, Topology of Metric Space, 2nd Ed, Narosa publication, 2011, ISBN-9788184870589

MMAC 1.41
ABSTRACT ALGEBRA

Theory Credit: 5
Tutorial Credit: 1

UNIT I A brief review of groups, their elementary properties and examples, subgroups, cyclic groups, homomorphism of groups and Lagrange's theorem; permutation groups, permutations as products of cycles, even and odd permutations, normal subgroups, quotient groups; isomorphism theorems, correspondence theorem.

UNIT II Group action; Cayley's theorem, group of symmetries, dihedral groups and their elementary properties; orbit decomposition; counting formula; class equation, consequences for p-groups; Sylow's theorems.

UNIT III Applications of Sylow's theorems, conjugacy classes in S_n and A_n, simplicity of A_n. Direct product; structure theorem for finite abelian groups; invariants of a finite abelian group (Statements only)

UNIT IV Basic properties and examples of ring, domain, division ring and field; direct products of rings; characteristic of a domain; field of fractions of an integral domain; ring homomorphisms; ideals; factor rings; prime and maximal ideals, principal ideal domain; Euclidean domain; unique factorization domain.

UNIT V A brief review of polynomial rings over a field; reducible and irreducible polynomials, Gauss' theorem for reducibility of $f(x) \in Z[x]$;Eisenstein's criterion for irreducibility of $f(x) \in Z[x]$ over **Q**, roots of polynomials; finite fields of orders 4, 8, 9 and 27 using irreducible polynomials over $\mathbf{Z}_2$ and $\mathbf{Z}_3$.

Recommended Books and References:

1. P.B. Bhattacharya, S. K. Jain and S. R. Nagpal, Basic Abstract Algebra, 2000, 3rd edition Cambridge University Press, ISBN 978-0-521-54548-8
2. N. Jacobson,Basic Algebra I,2002, 3rd edition,Hindustan Publishing corporation, New Delhi, ISBN-13: 978-0-486-47189-1, ISBN-10: 0-486-47189-6
3. J. A. Gallian,Contemporary Abstract Algebra, 1999, 4th edition, Narosa Publishing House, New Delhi, ISBN 978-81-7319-269-2.
4. I. N. Herstein,Topics in Algebra, 2016, 2nd edition,Wiley India Pvt. Ltd, ISBN 978-81-265-1018-4
5. J. B. Fraleigh,A First Course in Abstract Algebra, 2002, 7th edition, Pearson Education Inc.,ISBN 978-81-7758-900-9
6. D.S. Dummit, R.M. Foote, Abstract Algebra,2003, 2nd edition, Wiley India Pvt. Ltd., ISBN 978-81-265-1776-3

SEMESTER – II

MMAC 2.11
NUMERICAL ANALYSIS

Theory Credit: 4

UNIT I **Introduction:** Algebraic and transcendental equations and their roots, direct and iterative methods, errors, truncation, initial approximations, error analysis, rate of convergence, algorithms.

Transcendental and polynomial equations: Bisection method. Methods based on first degree equation – secantmethod, regula-falsi method. Methods based on second degree equation – Newton-Raphson method, Muller method, Chebyshev method, multipoint iteration methods.

UNIT II **System of linear algebraic equations and eigenvalue problems:** Direct methods – Cramer's rule, Gauss elimination method, Gauss-Jordanelimination method, triangularization method, Cholesky method, partition method.Iteration methods – Gauss-Jacobi iteration method, Gauss-Seidel iteration method.Eigenvalue problems – power method, inverse power method.

UNIT III **Interpolation and approximation:** Lagrange interpolation, Newton's divided difference interpolation, finite difference operators, relation between differences and derivatives, Gregory-Newton forward and backward difference interpolations, Stirling and Bessel interpolations, Hermite interpolation, piecewise and spline interpolations.

UNIT IV **Differentiation:** Numerical differentiation, methods based on interpolation, methods based on finite differences.

Integration: Numerical integration, methods based on interpolation, Newton-Cotes methods, methods based on undetermined coefficients, Gauss-Legendre integration methods

UNIT V **Ordinary differential equation:** Initial value problems, Picard method, Euler method, backward Euler method, mid-point method.Single-step methods-Taylorseries method, Runge-Kutta method.Multistep methods-Adam-Bashforth methods, Nyström methods, Adams-Moulton methods, Milne-Simpson methods.

Recommended Books and References:

1. M. K. Jain, S. R. K. Iyenger and R. K. Jain, *Numerical Methods for Scientific and Engineering Computation*, 6th edition, New Age International Publisher, India, 2007, ISBN-9788122414615
2. V. Rajaraman, *Computer Oriented Numerical Methods*, 4th Edition, Prentice Hall India Prt. Ltd., 2018, ISBN-9789388028318
3. Kendall Atkinson, *An Introduction to Numerical Analysis*, 2nd edition, John Wiley & Sons, 1989, ISBN-9780471624899
4. F. B. Hildebrand, *Introduction to Numerical Analysis*, 2nd edition, Dover Publications Inc., 1987, ISBN-9780486653631

MMAC 2.12
PROGRAMMING IN C (PRACTICAL)

Practical Credit: 2

Flowchart and algorithms, character set, identifiers, keywords, data types, constants and variables, statements, expressions, operators, precedence of operators, input-output, assignments, control structures, decision making and branching, decision making & looping, user-defined and standard functions, formal and actual arguments, functions category, function prototypes, parameter passing, call-by-value, call-by-reference, recursion, storage classes, one-dimensional array, multidimensional array declaration and their applications.

C programs to perform the following: (real solution only)

1. Solving simplealgebraic and transcendental equations using bisection method
2. Solving simplealgebraic and transcendental equations using secant method
3. Solving simplealgebraic and transcendental equations using regula-falsi method
4. Solving simplealgebraic and transcendental equations using Newton-Raphson method
5. Solutions of system of linear equations using Gauss elimination method.
6. Solutions of system of linear equations using Gauss-Seidel iteration method.
7. Matrix inversion using Gauss elimination method.
8. Matrix inversion using Gauss-Jordan method.
9. Power method for finding dominant eigenvalue.
10. Lagrange interpolation
11. Newton's divided difference interpolation
12. Numerical differentiation using Lagrange's formula.
13. Numerical differentiation using Newton's formula.
14. Numerical integration using trapezoidal rule.
15. Numerical integration using Simpson's rule.
16. Numerical solutions of ordinary differential equations (initial value problems) using Euler-Richardson method.
17. Numerical solutions of ordinary differential equations (initial value problems) using Runge-Kutta method.
18. Numerical solutions of ordinary differential equations (initial value problems) using predictor-corrector method.

Recommended Books and References:

1. V. Rajaraman, *Computer Programming in C*, Prentice Hall India Prt. Ltd., 1994, ISBN-9788120308596
2. V. Rajaraman, *Computer Oriented Numerical Methods*, 4th Edition, Prentice Hall India Prt. Ltd., 2018, ISBN-9789388028318
3. Brian W. Kernighan and Dennis M. Ritchie, *The C Programming Language*, 2nd edition, Pearson Education, Inc., 1988, ISBN-9789332549449
4. M. K. Jain, S. R. K. Iyenger and R. K. Jain, *Numerical methods for Scientific and Engineering Computation*, 6th edition, New Age International Publisher, India, 2007, ISBN-9788122414615

MMAC 2.21
GENERAL TOPOLOGY

Theory Credit: 5
Tutorial Credit: 1

UNIT 1 Topological spaces: Open sets, closed sets, neighborhoods, limit points, interiors, closures, boundary points, bases, sub bases. Finer and coarser topology, subspace topology, order topology, product topology, metric topology.

UNIT 2 Continuity: Continuous functions, continuity at a point, Sequential continuity at a point, open functions, closed functions, Homeomorphisms, Composition of continuous functions, Pasting lemma. Quotient topology, Quotient spaces.

UNIT 3 Connectedness: Connected spaces. Connected subspaces of the real line. Intermediate value theorem. Path connected space. Components and path components. Local connectedness.

UNIT 4. Compactness: Covers. Compact sets. Subsets of a compact space. Finite intersection properties. Compactness and Hausdorff spaces. Compact sets in the real line. Limit point compactness. Sequential compactness. Local compactness. One-point compactification. Tychonoff Theorem

UNIT 5 Countability and separation axioms: First countable spaces. Second countable spaces.Lindelöf space.Separable spaces. Hereditary properties. T1 spaces. Hausdorff spaces. Regular spaces. Normal spaces. The Urysohn lemma. Uryshon metrization Theorem, Tietze extension theorem. Stone- Cech Compactification.

In all units a good number of examples need to be discussed.

Recommended Books and References:

1. James R Munkres,Topology, Pearson Education India
2. K.D. Joshi, Introduction to General Topology, New Age International Pvt Ltd.
3. George F. Simmons, Introduction to Topology and Modern Analysis, McGraw Hill Ed.
4. Stephen Willard,General Topology, Dover publication

MMAC 2.31
CLASSICAL MECHANICS

Theory Credit: 5
Tutorial Credit: 1

UNIT I Basic concepts; Constraints; Generalized coordinates; Principle of virtual work; D' Alembert's principle; Lagrange's equations from D' Alembert's principle; Non-conservative forces; Generalized potential; Hamilton's principle and Lagrange's equations; Gauge invariance of the Lagrangian; Symmetry properties of space and time; Conservation laws; Invariance under Galilean transformation.

UNIT II Generalized momentum; Cyclic coordinates; Conservation theorems; Hamiltonian function and conservation of energy; Jacobi's integral; Hamilton's equations; Hamilton's equations in different coordinate systems; Harmonic oscillator; Motion of a particle in a central force field; Charged particle moving in an electromagnetic field; Compound pendulum; Two-dimensional harmonic oscillator;

Routhian.

UNIT III Calculus of variations; Euler-Lagrange's equations; Hamilton's principle from D' Alembert's principle; Modified Hamilton's principle; Deduction of Hamilton's principle from modified Hamilton's principle; Deduction of Lagrange's equations from variational principle for non-conservative systems (holonomic constraints); Lagrange's method of undetermined multipliers; Lagrange's equations of motion for non-holonomic systems; Physical significance of Lagrange's multipliers □;□-variation; Principle of least action; Jacobi's form of principle of least action.

UNIT IV Canonical transformations; Legendre transformations; Generating functions; Procedure for application of canonical transformations; Condition for canonical transformation; Bilinear invariant condition; Integral invariant of Poincare; Infinitesimal contact transformation;Poisson's bracket; Lagrange's bracket; Relation between Lagrange's and Poisson's brackets; Angular momentum and Poisson brackets; Invariance of Poisson bracket with respect to canonical transformations; Phase space; Liouville's theorem.

UNIT V Generalized coordinates of a rigid body; Body and space reference systems; Euler's angles; Infinitesimal rotations as vectors; Angular velocity; Components of angular velocity; Angular momentum and inertia tensor; Principle axes-Principle moments of inertia; Rotational kinetic energy of a rigid body; Symmetric bodies; Moments of inertia for different body systems; Euler's equations of motion for a rigid body; Torque-free motion of a rigid body; Force-free motion of a symmetrical top; Motion of a heavy symmetrical top; Fast top; Sleeping top; Gyroscope.

Recommended Books and References:

1. H. Goldstein, Classical Mechanics, Addison Wesley Publications, Massachusetts, 2002.
2. J. C. Upadhyaya, Classical Mechanics, Himalaya Publishing House, Mumbai, 2018.
3. K. SankaraRao, Classical Mechanics, Prentice Hall of India, New Delhi, 2015.
4. C. R. Mondal, Classical Mechanics, Prentice-Hall of India, 2001.
5. T. W. B. Kibble, Classical Mechanics, Orient Longman, London, 1985.
6. L. D. Landau and E. M. Lipshitz, Mechanics, Pergamon Press, Oxford, 1976.
7. J. E. Marsden, Lectures on Mechanics, Cambridge University Press, 1992.

MMAC 2.41
COMPLEX ANALYSIS

Theory Credit: 5
Tutorial Credit: 1

UNIT I Convergence of sequences and series, Absolute and uniform convergence of power series, Integration and differentiation of power series, uniqueness of series representations.

UNIT II Taylor series, Zeros of analytic functions, Limit points of Zeros, Singularities and their classification, Behaviour of the function in a neighbourhood of isolated singularities, Laurent's series, Residues, Cauchy Residue Theorem.

UNIT III Evaluation of improper integrals and definite integrals involving sines and cosines, integration through a branch cut.

UNIT IV The winding number, Logarithmic residues and Rouche'stheorem, the Argument Principle, Inverse Laplace Transforms.

UNIT V Mapping by elementary functions, Linear fractional transformations, cross ratios, mappings of the half planes and circles, conformal mapping, Statement of Riemann Mapping Theorem.

Recommended Books and References:

1. Mathews, J. H. and Howell, R. W., Complex Analysis for Mathematics and Engineering, 2010, 5th Edition, Narosa, ISBN-10: 817319761X ISBN-13: 978-8173197611
2. Conway, J. B., Functions of One Complex Variable, 1994, 2nd Edition, Narosa Publishing House, India, ISBN 978-81-85015-37-8
3. Churchill, R. V. and Brown, J. W., Complex Variables and Applications, 2014, 8th edition, McGraw-Hill Education (India), ISBN-13: 978-93-392-0515-7, ISBN-10: 93-392-0515-4
4. Ahlfors, L. V., Complex Analysis, 1979, 3rd Edition, McGraw-Hill Education, ISBN-13: 978-1-25-906482-1, ISBN-10: 1-25-906482-4
5. Priestly, H.A., Introduction to Complex Analysis, 2003, 2nd Edition, Oxford University Press, ISBN-10: 0198525613, ISBN-13: 978-0198525615
6. Gamelin, T. W., Complex Analysis, 2006, 1st Edition, UTM, Springer-Verlag,.ISBN-10: 8181281144, ISBN-13: 978-8181281142
7. Narasimhan, R. and Nievergelt, Y., Complex Analysis in One Variable, 2001, 2nd Edition, Birkhäuser, ISBN-10: 1461266475, ISBN-13: 978-1461266471
8. Donald Sarasan, Complex Function Theory, 2008, 2nd Edition, Hindustan Book Agency, ISBN-10: 9788185931845, ISBN-13: 978-8185931845

SEMESTER – III

MMAC 3.11
PARTIAL DIFFERENTIAL EQUATIONS

Theory Credit: 4

UNIT I PDE of the first order; Origin; Surfaces and normal; Curves and their tangents; Formation of partial differential equation; Solution of PDE of first order; Integral surfaces passing through a given curve; The Cauchy problem. Surfaces orthogonal to a given system of surfaces; First order non-linear equations – Cauchy method of characteristics; Compatible systems of first order equations; Charpit's method; Special types of first order equations.

UNIT II Origin of second order PDE; Classification of second order PDE; Canonical forms; Adjoint operators; Riemann's method; Linear second order PDE with constant coefficients – General method for finding complementary functions of (i) reducible non-homogeneous linear PDE (ii) irreducible non-homogeneous linear PDE; Methods for finding particular integrals; Homogeneous linear second order PDE with constant coefficients – methods for finding complementary functions and particular integrals; Linear second order PDE with variable coefficients; Monge's method of solution of non-linear PDE of second order.

UNIT III Occurrence of the Laplace and Poisson equations; Derivation of Laplace and Poisson equations; Boundary value problems; Properties of harmonic functions; The spherical mean; Mean value theorem for harmonic functions; Maximum-Minimum principle and consequences; Separation of variables; Dirichlet problem for a rectangle; The Neumann problem for a rectangle; Interior and exterior Dirichlet problem for a circle; Interior Neumann problem for a circle; Solution of Laplace's equation in cylindrical and spherical coordinates.

UNIT IV Occurrence of diffusion equation; Boundary conditions; Elementary solutions of diffusion equation; Dirac-Delta function; Separation of variables; Solution of diffusion equation in cylindrical and spherical coordinates; Maximum-Minimum principle and consequences; Non-linear equations-semi-linear, quasi-linear and Burger's equations; Initial value problem for Burger's equation.

UNIT V Occurrence of the wave equations; Derivation of one-dimensional wave equation; Solution of one-dimensional wave equation by canonical reduction; The initial value problem – D' Alembert's solution; Vibrating string-variables separable solution; Forced vibration; Boundary and initial value problem for two-dimensional wave equations – method of Eigen function; Periodic solution of one-dimensional wave equation in cylindrical and spherical polar coordinates; Vibration of circular membrane; Uniqueness of solution for the wave equation; Duhamel's principle.

Recommended Books and References:

1. I. N. Sneddon, Elements of Partial Differential Equations (3rd edition), McGraw Hill Book Company, 1998
2. K. Sankara Rao, Introduction to Partial Differential Equations, Prentice Hall of India, 2017
3. I. N. Sneddon, Elements of Partial Differential Equation (3rd edition), McGraw Hill Book Company, 1998.
4. E. T. Copson, Partial Differential Equations (2nd edition), Cambridge University Press, 1995.
5. Tyn Myint-U & Lokenath Debnath, Linear Partial Differential Equations for Scientists and Engineers, Birkhauser, 2007.

MMAC 3.12
PARTIAL DIFFERENTIAL EQUATIONS (PRACTICAL)

Practical Credit: 2

Problems from Partial Differential Equations (Theory) may be solved with the help of software like MAPLE / MATHEMATICA / MATLAB/ any open source software.

MMAC 3.21
FUNCTIONAL ANALYSIS

Theory Credit: 5
Tutorial Credit: 1

UNIT I General Banach space – Definition and Examples, continuous linear transformations between Normed linear spaces, Riesz Lemma, Hahn – Banach theorem and its consequences.

UNIT II Classical Banach space, L^p spaces, Holder's inequality, Minkowski inequality, Convergence and completeness, Riesz – Fischer theorem Subspace and Quotient spaces of Banach spaces, Riesz representation theorem.

UNIT III Embedding of a Normed linear space in its second conjugate space, open mapping theorem, closed graph theorem, uniform boundedness theorem, conjugate of an operator.

UNIT IV Hilbert's space , example and simple properties, orthogonal complement, orthonormal set, Bessel's inequalities, complete orthonormal set, Gram-Schmidt orthogonalization process, self adjoint operators.

UNIT V Normal and unitary operators, Projections, spectrum of an operator, spectral theorem for a normal operator on a finite dimensional Hilbert space.

Recommended Books and References:

1. H. L. Royden, Real Analysis (4th edition), Macmillan Publishing co. inc. New York, 2005, ISBN-9789332551589
2. G. F. Simmons, Introduction to topology and Modern Analysis (4th edition), Tata McGraw- Hill Ltd, 2017, ISBN-9780070597846
3. W. Rudin,Functional Analysis, Tata McGraw hill, 1974, 2nd Ed. ISBN-9780070619883
4. B. V. Limaye, Functional Analysis, Willy Eastern ltd., 1991, 3rd Ed, ISBN-9789386286093
5. J B Conway, A course in functional analysis, Springer, 1997, 2nd Ed, ISBN-9783540960423

SEMESTER – IV

MMAC 4.11
MATHEMATICAL METHODS

Theory Credit: 4

UNIT I Laplace transforms properties of Laplace transform; inversion formula convolution; application to ordinary and partial differential equations; Fourier transform; properties of Fourier transform; inversion formula, convolution; Parseval's identity; Fourier transform of generalized functions; application of transforms to heat, wave and Laplace equation.

UNIT II Formulation of integral equations; integral equations of Fredholm and Volterra type; solution by successive substitution and successive approximation; integral equations with degenerate kernels.

UNIT III Integral equations of convolution type and their solutions by Laplace transform; Fredholm's theorems; integral equations with symmetric kernel; eigenvalues and eigenfunctions of integral equations and their simple properties.

UNIT IV Mikusinski's operational calculus of one variable (algebra of addition and convolution of functions, ordered pairs of functions, convolution quotients of a function with a nonzero function); Dirac delta function.

UNIT V Eigenvalue problem; ordinary differential equations of the Sturm-Liouville type; eigenvalues and eigenfunctions; expansion theorem; extrema properties of the eigenvalues of linear differential operators; formulation of the eigenvalue problem of a differential operator as a problem of integral equation. Green’s Function.

Recommended Books and References:

1. Michael Gambier Smith, Laplace Transform Theory, D. Van Nostrand Company Ltd., London, 2000.
2. Georgi. E. Shilov& Bernard Seckler, Generalized Functions and Partial Differential Equations, Gordon and Breach Science Publisher Inc., 1999.
3. M. Krasnov, A. Kiselev, G. Makarenko, Problems And Exercises In Integral Equations, Mir Publishers, 1971
4. David Porter and David S. G. S, Integral Equations, Cambridge University Press, 1993.
5. I. N. Sneddon, The Use of Integral Transforms, Tata McGraw Hill, New Delhi, 1974.
6. R. R. Goldberg L, Fourier Transforms, Cambridge University Press, 1970.
7. H. Widom, Lectures on integral equations, Van Nast Rand, 1969.

MMAC 4.12
MATHEMATICAL METHODS (PRACTICAL)

Practical Credit: 2

By using any mathematical software

1. Laplace Transforms.
2. Inverse Laplace Transforms.
3. Solution of Ordinary Differential Equation by Laplace Transforms.
4. Solution of Partial Differential Equation by Laplace Transforms.
5. Solving Initial Value Problem with Laplace Transforms.

6. The DiracDelta Function.
7. Convolution.
8. Solution of Integral Equation.
9. Fourier Transforms.
10. Inverse Fourier Transforms.
11. Plotting a Fourier Transform.
12. Integro-Differential Equations.
13. Boundary Value Problem.
14. Sturm-Liouville Eigenvalue Problems.
15. One-Dimensional Heat Equation.
16. Diffusion Problems over Infinite and Semi-infinite Domains.
17. Sturm-Liouville Problem for the Wave Equation.

Recommended Books and References:

1. Martha L. Abill and Jamis P. Brasilton, Differential Equations with Maple V®, Academic Press, Inc., London, 1962
2. George A. Articolo, Partial Differential Equations and Boundary Value Problems with Maple, Second Edition, Elsevier Inc., 2009

MMAC 4.21
RINGS AND MODULES

Theory Credit: 5
Tutorial Credit: 1

UNIT I Basic concepts of Rings, Division rings and Fields, Algebra of Ideals, Matrix Rings, Local Rings, Opposite Rings, Direct product of Rings, Endomorphism Rings, Embedding of Rings, Idempotent and Nilpotent elements in a ring.

UNIT II Modules, Submodules, Quotient Modules, Module Homomorphism, Isomorphism Theorems; Exact sequences, the group of homomorphisms and its properties relative to exact sequences.

UNIT III Direct sums and Direct products of Modules, External and Internal direct sums, Direct summands, Zorn's lemma, Free modules and Projective modules, Torsion free and Torsion modules over commutative domains, Exact sequences and Projectivity.

UNIT IV Injective modules, Injectivity and Divisibility over domains, Exact sequences and Injectivity; Baer's theorem and its elementary applications; Simple modules, Semisimple modules (as per Bourbaki); Schur's lemma.

UNIT V ArtinianModules, Noetherian Modules, Modules of Finite Length, Nil Radical and Jacobson Radical, Simple Rings, Semisimple Rings, Artinian Rings, Noetherian Rings.

Recommended Books and References:

1. C. Musili, Introduction to Rings and Modules, 2015, 2ndRevised Edition, Narosa Publishing House, ISBN 978-81-7319-037-2
2. I. T. Adamson, Elementary Rings and Modules, 1995, Oliver and Boyd, Edinburgh, ISBN-10: 0050021923, ISBN-13: 978-0050021927
3. J.J. Rotman, Notes on Homological Algebra, 1990, Van Nostrand Reinhold Inc., ISBN-10: 0442270607, ISBN-13: 978-0442270605
4. N. Jacobson, Basic Algebra II, 2002, 3rd Edition, Hindustan Publishing Corporation,New Delhi, ISBN-10: 071671079X, ISBN-13: 978-0716710790

5. S. Lang, Algebra, 1984, 2nd Edition, Addison-Wesley, Massachusetts, ISBN-10: 0201555409, ISBN-13: 978-0201555400
6. I. S. Luthar an d I.B.S. Passi, Algebra, Vol. 2: Rings, 1999, Narosa Publishing House, New Delhi, ISBN 978-81-7319-313-2
7. D.S. Dummit, R.M. Foote, Abstract Algebra, 2003, 2nd edition, Wiley India Pvt. Ltd., ISBN 978-81-265-1776-3

DISCIPLINE SPECIFIC ELECTIVE 1 & 2
MMAD 3.11 & 3.21

NUMBER THEORY

Theory Credit: 5
Tutorial Credit: 1

UNIT I Review of Divisibility–Division algorithm, greatest common divisor, Euclidean algorithm, primes, fundamental theorem of arithmetic.
Review of Congruences–Basic properties of congruences, complete residue system, reduced residue system, Euler's phi-function, Fermat's theorem, Euler's theorem, Wilson's theorem, solutions of congruences, Chinese remainder theorem, prime power moduli, prime modulus.

UNIT II Primitive roots and power residues, Euler's criterion, Quadratic residues, Legendre symbol, Gauss's lemma, quadratic reciprocity, Jacobi symbol, binary quadratic forms.

UNIT III Diophantine equations**-**The equation $ax + by = c$, simultaneous linear equations, Pythagorean triples, sum of squares, assorted examples.

UNIT IV Farey fractions irrational numbers: Farey sequences, rational approximations, irrational numbers. Recurrence functions, Fibonacci sequence, identities involving Fibonacci numbers.

UNIT V Simple continued fractions-Finite and infinite continued fractions, uniqueness, expansion of irrational numbers as infinite simple continued fractions, approximations to irrational numbers, Hurwitz theorem, periodic continued fractions, Pell's equation.

Recommended Books and References:

1. Ivan Niven, Herbert S. Zuckerman, and Hugh L. Montgomery, *An Introduction to the Theory of Numbers*, 5thEdition, John Wiley & Sons, 1991, ISBN-9788126518111
2. David M. Burton, *Elementary Number Theory*, 6th Edition, Tata McGraw-Hill, 2007, ISBN-9781259025761
3. G. H. Hardy, Edward M. Wright, and Roger Heath-Brown, *An Introduction to the Theory of Numbers*, 6thEdition, Oxford University Press, 2008, ISBN-9780199219865

OPERATIONS RESEARCH

Theory Credit: 5
Tutorial Credit: 1

UNIT I **OR Fundamentals**: Introduction to Operations Research: Basic definition, scope, objectives, phases, models and limitations of Operations Research. Linear Programming Problem – Formulation of LPP, Graphical solution of LPP. Simplex Method, Artificial Variables, big-M method, two-phase method, degeneracy and unbounded solutions, Sensitivity Analysis-Graphical Approach.

UNIT II **Non-Linear Programming:** Non-Linear Programming: Single Variable Optimization, Sequential Search Techniques, Fibonacci search, Convex functions, Multi-variable Optimizations without Constraints: the method of Steepest Ascent, Newton-Raphson method, Multi-Variable Optimizations with Constraints: Lagrange

Multipliers, Newton-Raphson's method, Penalty Functions, Kuhn-Tucker conditions.

UNIT III **Deterministic Inventory Modeling:** Introduction to inventory systems, Selective inventory Classification and its use in controlling inventory. Deterministic inventory models: Economic Order Quantity (EOQ) model, EOQ with finite supply, EOQ with backorders, EOQ with constraints, All-units quantity discounts model.

UNIT IV **Network Analysis:** Networks, Minimum-span problems, Shortest route problems, Maximal flow problems, PERT/CPM. Critical path computations for PERT, Construction of Time schedules.LPP formulations for PERT.

UNIT V **Game Theory:**Concepts of Game problem. Two-person zero sum game.Pure and Mixed strategies.Competitive games, rectangular games, saddle point, minimax (maximin) methods of optimal strategies, value of the game. Solution of games with saddle points, dominance principle. Rectangular games without saddle point-mixed strategy for 2x2 games.

Recommended Books and References:

1. Hamdy A. Taha: Operations Research – An Introduction, Pearson Education,2007
2. F.S.Hillier, G.J.Lieberman : Introduction to Operations Research – Concepts and Cases, 9th edition, Tata McGraw Hill – 2010
3. J.K Sharma: Operations Research Theory and applications,Trinity Press , 6th edition
4. N.V.S Raju, Operations Research, HI-TECH,2002

TENSOR ANALYSIS AND RIEMANNIAN GEOMETRY

Theory Credit: 5
Tutorial Credit: 1

UNIT I Contravariant and Covariant vectors, Tensor, tensor field, addition, subtraction and multiplication of tensors, contraction, symmetric tensor, antisymmetric tensor, quotient law, reciprocal symmetric tensor, relative tensor, relative vector and relative scalar.

UNIT II Riemannian metric, fundamental tensor, length of curve, magnitude of vector, associated covariant and contravariant vectors, inclination of two vectors, orthogonal vectors coordinate hypersurfaces, coordinate curves, field of normals to hypersurface, principal directions for a symmetric covariant tensor of the second order, Euclidean space of n – dimension.

UNIT III Christoffel symbols, tensor law of transformation of Christoffel symbols, covariant derivative of covariant and contravariant vectors, covariant derivative of a covariant second rank tensor, divergence and curl of a vector, theorem related to divergence.

UNIT IV Curvature of a curve, geodesic, differential equation of a geodesic, Riemannian coordinates geodesic form of line element, geodesic in Euclidean space, Levi-Civita's concept of parallelism, Subspaces of Riemannian manifold, Fundamental theorem of Riemannian geometry.

UNIT V Curvature tensor, Bianchi identities, covariant curvature tensor, theorem on Riemannian curvature, Schur's theorem, Wily tensor (projective tensor) mean curvature, Ricci principal directions, Einstein space, Ricci coefficient of rotation and its properties, curvature of congruence, Geodesic of congruence, curl of congruence, canonical congruence.

Recommended Books and References:

1. C. E. Weatherburn, An introduction to Riemannian Geometry and Tensor Calculus, Cambridge university press, 1986, ISBN-9780521091886
2. Leonor Godinho and JoseNatario, An Introduction to Riemannian Geometry, Springer nature, 2014, ISBN-9783319086651
3. Nail H. Ibragimov,Tensor and Riemannian geometry, De Gruyter, 2015, ISBN-9783110379648
4. T J Wilmore, An Introduction to Differential Geometry, Oxford University Press, 32nd impression, 2017, ISBN-9780195611106

MEASURE THEORY

Theory Credit: 5
Tutorial Credit: 1

UNIT I SETS AND CLASSES: Set inclusion, Unions and intersections, Limits, complements, and differences, Rings and algebras, Generated rings and sigma-rings, Monotone classes

UNIT II MEASURES AND OUTER MEASURES: Measure on rings, Measure on intervals, Properties of measures, Outer measures, Measurable sets

UNIT III EXTENSION OF MEASURES: Properties of induced measures, Extension, completion, and approximation, Inner measures, Lebesgue measure, Non measurable sets

UNIT IV MEASURABLE FUNCTIONS: Measure spaces, Measurable functions.Combinations of measurable functions, Sequences of measurable functions, Pointwise convergence, Convergence in measure.

UNIT V INTEGRATION: Integrable simple functions, Sequences of integrable simple functions, Integrable functions. Sequences of integrable functions, Properties of integrals.L^p Spaces.

Recommended Books and References:

1. G. De Barra, Measure Theory and Integration, 1981. E Horwood publisher
2. Paul R Halmos, Measure Theory, 2008, Springer
3. Terence Tao, An Introduction to Measure Theory,
4. Robert G Bartle, The Elements of Integration and Lebesgue Measure
5. G B Folland, Real Analysis, Modern Techniques and their applications

GRAPH THEORY

Theory Credit: 5
Tutorial Credit: 1

UNIT I Graphs, Incidence and degree, Isomorphism, Sub graphs, walks, path & circuits, connected graphs, disconnected graphs and component, Euler graphs, Fluery's algorithm, Hamiltonian paths and circuits, the Chinese Postman Problem, the traveling salesman problem, Matrix representation of graph: Incidence matrix, Circuit matrix, Path matrix, Cut-set matrix and Adjacency matrix.

UNIT II Shortest Path Algorithms: Dijkstra's algorithm, Floyd-Warshall algorithm, Trees, distance diameters, radius and pendent vertices, rooted and binary trees, counting trees, spanning trees, finding all spanning trees of a graph and a weighted graph, Prim's and Kruskal Algorithms for minimal spanning tree.

UNIT III Cut sets and cut vertices, fundamental circuits and cut sets, network flows, 1-Isomorphism, 2-Isomorphism, Planar graphs, Kuratowski two graphs, detection of planarity, geometric dual, criterion of planarity, thickness and crossings.

UNIT IV Coloring, covering and partitioning of a graph, chromatic number, chromatic partitioning, chromatic polynomials, matching, covering, four color problem.

UNIT V Digraphs and binary relations, Euler digraphs, Vector and Vector space of a graph, basis vector, cut set vector, circuit vector, circuit and cut set subspaces, Orthogonal Vectors and Spaces

Recommended Books and References:

1. Harary. F, Graph Theory, Narosa Publishing House, 2001. ISBN-978-8185015552
2. Bondy and Murthy, Graph theory and application, Addison Wesley, 2011, ISBN-978-1447173601
3. Deo N, Graph theory with applications to Engineering and Computer Science, PHI Learning, 2009, ISBN-978-8120301450
4. V Balakrishnan, Schaum's Outline of Graph Theory: Including Hundreds of Solved Problems, Mcgraw Hill, 1997, ISBN-978-0070587182

MATHEMATICAL STATISTICS

Theory Credit: 5
Tutorial Credit: 1

UNIT I The postulates of probability, Some elementary theorems, addition and multiplication rules, Bayes rule and future Bayes theorem, random variables, mathematical expectation, probability mass functions and probability density function, distribution function and its properties.

UNIT II Uniform, Bernoulli and binomial distribution, hypergeometric and geometric distribution, negative binomial and Poisson distribution, uniform and exponential distribution, gamma andbeta distributions, normal distribution, log-normal distribution.

UNIT III Moments and moment generating functions, probability generating functions and characteristic function Moments of binomial, hypergeometric, Poisson, gamma, beta and normal distributions, transformation of variables: one variable, Several variables.

UNIT IV The distribution of sample moments, the distribution of differences of means and variances, the Chi-Square distribution, the t distribution and the F distribution.

UNIT V Multiple linear regression, estimation of parameters using method of least square, use of dummy variables, binomial logistic regression and multinomial logistic regression- estimation of the regression coefficients and their interpretation.

Recommended Books and References:

1. J. E. Freund, Mathematical Statistics, Prentice Hall Inc., 1992
2. Bhattacharjee D. and Das K. K., A Treatise on Statistical Inference and Distributions, Asian Books, 2010
3. Hogg and Craig, Introduction to Mathematical Statistics, Collier Macmillan, 1958
4. Mood, Greyill and Boes, Introduction to the Theory of Statistics, McGraw Hill
5. R. E. Walpole, Introduction to Statistics, Macmillan Publishing Company, 1982
6. M. R. Spiegel and L. J. Stephens, Statistics, McGraw Hill Book Company, 1984

FIELD THEORY

Theory Credit: 5
Tutorial Credit: 1

UNIT I Extension Fields, Finite Extensions, Algebraic and Transcendental Elements, Adjunction of Algebraic Elements, Kronecker Theorem, Algebraic Extensions, Splitting Fields- Existence and Uniqueness, Extension of Base Field Isomorphism to Splitting Fields.

UNIT II Simple and Multiple Roots of Polynomials, Criterion for Simple Roots, Separable and Inseparable Polynomials, Perfect Fields, Separable and Inseparable Extensions, Finite Fields, Prime Fields and Their Relations to Splitting Fields, Frobenius Endomorphism, Roots of Unity and Cyclotomic Polynomials

UNIT III Algebraic Closed Fields and Algebraic Closures, Primitive Element Theorem, Normal Extensions, Automorphism Groups and Fixed Fields, Galois Pairing, Determination of Galois Groups, Fundamental Theorem of Galois Theory, Abelian and Cyclic Extensions.

UNIT IV Normal and Subnormal Series, Composition Series, Jordan-Holder Theorem (Statement Only), Solvable Groups, Nilpotent Groups.

UNIT V Solvability by Radicals, Solvability of Algebraic Equations, Symmetric Functions, Ruler and Compass Constructions, Fundamental Theorem of Algebra.

Recommended Books and References:

1. P. B. Bhattacharya, S. K. Jain and S. R. Nagpal, Basic Abstract Algebra, Cambridge University Press
2. N. Jacobson, Basic Algebra I, Hindustan Publishing Corporation, New Delhi
3. Galois Theory, TIFR Mathematical Pamphlets, No. 3, 1965
4. I. N. Herstein, Topics in Algebra, Wiley Eastern Limited, New Delhi
5. J. B. Fraleigh, A Frist Course in Abstract Algebra, Narosa Publishing House, New Delhi
6. J. A. Gallian, Contemporary Abstract Algebra
7. Gopalakrishnan, University Algebra

MATHEMATICAL MODELING

Theory Credit: 5
Tutorial Credit: 1

UNIT I Mathematical modeling introduction, techniques, classifications, some illustrations: mathematical modeling through geometry/ algebra/ trigonometry/ calculus, mathematical modeling through ODE of first order: linear growth and decay model, non-linear growth and decay model, compartment models mathematical modeling of dynamics, geometrical problem.

UNIT II Mathematical modeling through systems of ordinary differential equations of first order: in population dynamics, epidemics, economics, medicine, dynamics, mathematical modeling through ODE of second order: of planetary motions and motion of satellites, modeling through linear ordinary differential equations of second order in electrical circuits, catenary.

UNIT III Mathematical modeling through difference equations with constant coefficients: in population dynamics and genetics, mathematical modeling through PDE: mass-balance equations, momentum balance equations, variational principles, model for traffic on a highway.

UNIT-IV Mathematical modeling through graphs: in terms of directed graphs in terms of signed graphs, in terms of weighted diagraphs and in terms of unoriented graphs.

UNIT-V Mathematical modeling through linear programming: of different industrial oriented problems, mathematical modeling through calculus of variations: on geometrical problems, problems of mechanics/ bioeconomics.

Recommended Books and References:

1. Kapur, J.N., Mathematical modeling, New Age International
2. Burghes, D.N., Mathematical modeling in social, management and life sciences, ElliosHorwood and John Wiley
3. Giordano, F.R., and Weir, M.D., A first course in Mathematical Modeling, Brooks Cole
4. Kapur, J.N., Insight into mathematical modeling, Indian National Science academy
5. Bellomo and Preziosi, Modeling Mathematical methods and Scientific computation, CRC

MULTIVARIABLE CALCULUS

Theory Credit: 5
Tutorial Credit: 1

UNIT I Functions on Euclidean spaces, differentiability in several variables, partial and directional derivatives, chain rule, mean value theorem.

UNIT II Inverse function theorem, implicit function theorem, rank theorem, determinants, derivatives of higher order, differentiation of integrals.

UNIT III Riemann integration in higher dimensions, Fubini's theorem, change of variables, improperintegrals, line and surface integrals, Green's theorem, divergence theorem, Stokes' theorem.

UNIT IV Tensors, wedge product, differential forms, Poincare's lemma, integration on chains, Stokes' theorem for integrals of differential forms on chains, fundamental theorem of calculus.

UNIT V Differentiable manifolds (as subspaces of Euclidean spaces), differentiable functions on manifolds, tangent spaces, differential forms on manifolds, orientations, integration on manifolds, Stokes'theorem on manifolds.

Recommended Books and References:

1. J.R. Munkres, *Analysis on Manifolds*, Westview Press, 1st edition, 1991.ISBN: 9780201315967
2. W. Rudin, *Principles of Mathematical Analysis*, McGraw-Hill, 3rd edition, 1984.ISBN: 9781259064784
3. M. Spivak, *Calculus on Manifolds, A Modern Approach to Classical Theorems of AdvancedCalculus*, Westview Press, 1st edition, 1971.ISBN: 9780805390216
4. G. B. Folland, *Advanced Calculus*, Pearson Education, 1st edition, 2002.ISBN: 9780130652652
5. Tom M. Apostol, *Mathematical Analysis*, Addison-Wesley Publishing Company, Inc., 2nd Edition, 1974. ISBN: 9788185015668

DISCIPLINE SPECIFIC ELECTIVE 3 & 4
MMAD 4.11 & MMAD 4.21

FLUIDMECHANICS

Theory Credit: 5
Tutorial Credit: 1

UNIT I Lagrangian and Eulerian methods of description; Governing equations of fluid motion; stream line; velocity potential; path line; circulation; equations of continuity in Lagrangian and Eulerian methods; equivalence of the two forms of equations of continuity; Boundary surface; acceleration; Euler's equations of motion; integrals of Euler's equations of motion; Lagrange's equations of motion; Cauchy's integrals; equation of energy.

UNIT II Motion in two dimensions; stream function; complex potential; source; sink and doublet; images in two dimensions; images of a source with regard to a plane; a circle and a sphere; image of a doublet; circle theorem; Theorem of Blasius.

UNIT III Vortex Motion; Helmholtz properties of vortices; velocity in a vortex field; motion of a circular vortex; Infinite rows of vortices; Karman Vortex Street.

UNIT IV Viscous fluid; Navier-Stokes equations; diffusion of vorticity; dissipation of energy; steady motion of a viscous fluid between two parallel planes; steady flow through cylindrical pipes; Reynolds number.

UNIT V Waves motion in a gas; speed of sound; equation of motion of a gas; subsonic; sonic and supersonic flows of a gas; isentropic gas flow; flow through a nozzle; shock formation; elementary analysis of normal and oblique shock waves.

Recommended Books and References:

1. S. W. Yuan, Foundation of Fluid Mechanics, Prentice-Hall, 1967
2. W. H. Besant & A. S. Ramsey, A Treatise of Hydromechanics (3rd edition), G. Bell and Sons Ltd, 1997.
3. Frank Chorlton, Text Book of Fluid Dynamics, D. Van Nostrand Company Ltd., London, 1990.
4. M.D. Raisinghania, Fluid Dynamics
5. Shanti Swarup, Fluid Dynamics

FOURIER ANALYSIS

Theory Credit: 5
Tutorial Credit: 1

UNIT I Fourier Series. Fourier coefficients: basic properties. Fourier series: summability in norm. summability at a point. Fourier coefficients in $\ell^1(\mathbb{Z})$ (or $f \in A(T)$, $\ell^2(\mathbb{Z})$ (or $f \in L^2(T)$). Maximal functions. Fourier summabilitypointwisea.e. (almost everywhere).

UNIT II Fourier series: convergence at a point. Norm convergence. Hilbert transform on $L^2(T)$. Calderon – Zygmund decompositions. Hilbert transform on $L^p(T)$. Application of interpolation.

UNIT III Fourier integrals. Fourier transforms: basic properties. Summability in norms. Fourier inversion when $\hat{f} \in L^1(\mathbb{R}^d)$. Fourier transforms in $L^2(\mathbb{R}^d)$

UNIT IV Fourier integrals.Summabilitya.e. norm convergence. Hilbert and Riesz transforms on $L^2(\mathbb{R}^d)$. Hilbert and Riesz transforms on $L^p(\mathbb{R}^d)$

UNIT V Fourier series and integrals. Band limited functions. Periodization and Poisson summation.Uncertainty principles.

Recommended Books and References:

1. Richard S Laugesan, Harmonic Analysis Lecture Notes
2. Korner T.A,A First Look at Fourier Analysis
3. Kenneth B Howell, Principles of Fourier Analysis
4. Murray Spiegel, Schaum's Outline of Fourier Analysis with Applications to Boundary Value Problems

ALGEBRAIC NUMBER THEORY

Theory Credit: 5
Tutorial Credit: 1

UNIT I Preliminaries from commutative Algebra: Basic definition. Ideals in products of rings.Noetherian rings. Noetherian Modules.Local rings.Rings of fractions.The Chinese remainder theorem. Review of Tensor products.

UNIT II Rings of Integers: First proof that the integral elements form a ring. Dedekind's proof that the integral elements from a ring. Integral elements. Review of bases of A-modules. Review of norms and traces. Review of bilinear forms. Discriminants. Rings of integers are finitely generated. Finding the ring of integers.Algorithms for finding the ring of integers.

UNIT III Dedekind Domains: factorization: Discrete valuation rings. Dedekind domains.Unique factorization of ideals.The ideal class group.Discrete valuations.Integral closures of Dedekind domains.Modules over Dedekind domains.Factorization in extensions.The primes that ramify.Finding factorizations.Examples of factorizations.Eisenstein extensions.

UNIT IV The Finiteness of the Class Number: Norms of ideals. Statement of the main theorem and its consequences.Lattices.Some calculus.Finiteness of the class number. Binary quadratic forms.

UNIT V The unit theorem.Statement of the theorem.Proof that U_K is finitely generated.Computation of the rank.*S*-units. Example: CM fields. Example: Real quadratic fields, Cubic fields with negative discriminant. Finding $\mu(K)$.Finding a system of fundamental units.Regulators.

Recommended Books and References:

1. J.S. Milne, Algebraic Number Theory
2. Pierre Samuel, Algebraic Theory of Numbers
3. N. Ram Murthy &Jody Esmonde, Problems in Algebraic Number Theory
4. Alaka S, & William K, Introductory Algebraic Number Theory

ANALYTIC NUMBER THEORY

Theory Credit: 5
Tutorial Credit: 1

UNIT I Arithmetical Functions and Dirichlet Multiplication: Introduction, the Mobius function $\mu(n)$ The Euler totient function $\phi(n)$, the relationship connecting ϕ and μ. A product formula for $\phi(n)$.The dirichlet product of arithmetical functions.Dirichlet inverse and Mobius inversion formula.The Mangoldt function $\Lambda(n)$. Multiplicative functions. Multiplicative functions and Dirichlet multiplication.The inverse of a completely multiplicative function.

UNIT II Liouville's function $\lambda(n)$. The divisor functions $\sigma_{\alpha}(n)$. Generalized convolutions.Formal power series.The Bell series of an arithmetical function.Bell series and Dirichlet multiplication.Derivatives of arithmetical functions.The Selberg identity.

UNIT III Averages of Arithmetical Functions: Introduction. The big oh notation.Asymptotic equality of functions.Euler's summation formula.Some elementary asymptotic formulas. The average order of d(n). The average order of the divisor functions. The average order of $\phi(n)$. An application to the distribution of lattice points visible from the origin. The average order of $\mu(n)$ and $\Lambda(n)$. The partial sums of a Dirichlet product. Applications to $\mu(n)$ and $\Lambda(n)$. Another identity for the partial sums of a Dirichlet product.

UNIT IV Some Elementary Theorems on the Distribution of Prime Numbers.Introduction.Chebyshev's functions $\psi(x)$ and $\vartheta(x)$. Relations connecting $\vartheta(x)$ and $\pi(x)$. Some equivalent forms of the prime number theorem. Inequalities for $\pi(n)$ and p_n. Shapiro's Tauberian theorem.Applications of Shapiro's theorem. An asymptotic formula for the partial sums $\sum_{p \le x} \frac{1}{p}$. The partial sums of the Mobius functions.Brief sketch of an elementary proof of the prime number theorem.Selberg's asymptotic formula.

UNIT V Dirichlet's theorem for primes of the form *4n – 1* and *4n + 1*.The plan of the proof of Dirichlet's theorem.Proof of Lemma 7.4, 7.5, 7.6, 7.7 and 7.8.(referApostol Tom M., Introduction to analytic number theory) Distribution of primes in arithmetic progressions.

Recommended Books and References:

1. Tom M. Apostol, Introduction to analytic number theory
2. Paul T Bateman, Harrold G. Diamond, Analytic Number Theory: Introductory Course

ALGEBRAIC TOPOLOGY

Theory Credit: 5
Tutorial Credit: 1

UNIT I Homotopy of paths, fundamental group of a topological space, fundamental group functor, homotopy of maps of topological spaces; homotopy equivalence; contractible and simply connected spaces; fundamental group of S^1, S^1 x S^1 etc.; degree of maps of S^1.

UNIT II Calculation of fundamental groups of S^n (n > 1) using Van Kampen's theorem (special case); fundamental group of a topological group; Brouwer's fixed point theorem; fundamental theorem of algebra; vector fields, Frobenius theorem on eigenvalues of 3 x 3 matrices.

UNIT III Covering spaces, unique lifting theorem, path-lifting theorem, covering homotopy theorem, applications; criterion of lifting of maps in terms of fundamental groups; universal coverings and its existence; special cases of manifolds and topological groups.

UNIT IV Simplicial and singular homology, reduced homology, Eilenberg-Steenrod axioms (without proof), relation between Π_1 and H_1; relative homology.

UNIT V Calculations of homology of S^n; Brouwer's fixed point theorem for $f : E^n \rightarrow E^n$ (n > 2) and its applications to spheres and vector fields; Meyer-Vietoris sequence and its application.

Recommended Books and References:

1. J. R. Munkres, Topology, a first course, Prentice- Hall of India Ltd., New Delhi, 2000.
2. M. J. Greenberg and J. R. Harper, Algebraic topology, a first course (2nd edition), Addison-Wesley Publishing co., 1997.
3. A. Hatcher, Algebraic Topology, Cambridge University Press, 2002.
4. E. H. Spanier, Algebraic Topology (2nd edition), Springer-Verlag, New York, 2000.
5. J. J. Rotma, An Introduction to Algebraic Topology, Graduate Text in Mathematics, No. 119, Springer, New York, 2004.
6. W. Fulton, Algebraic topology, a first course (2nd edition), Graduate Text in Mathematics, No. 153, Springer, New York, 1995.
7. S. Eilenberg and N. E. Steenrod, Foundations of Algebraic Topology (2nd edition), Princeton University Press, 1995.

DIFFERENTIAL GEOMETRY OF MANIFOLDS

Theory Credit: 5
Tutorial Credit: 1

UNIT I Differentiable Manifold, Differentiable functions, tangent space, vector fields, maps, exterior algebra, exterior derivative.

UNIT II Lie group and Lie algebra, one parameter subgroup and exponential maps, homomorphism and isomorphism, principal fiber bundle, associated fiber bundle, induced bundle, bundle homomorphism.

UNIT III Linear connections, parallelism, pseudo tensorial forms, basic form, torsion and curvature form, Expression in local coordinates covariant derivative, torsion and curvature tensors, geodesic and lie derivative.

UNIT IV Riemannian manifold: Riemannian metric, Riemannian connection, curvature tensor, sectional curvature, projective curvature tensor, conformal curvature tensor.

UNIT V Submanifolds, normals, induced connections, Gauss formulae, Weingerten formulae, lines of curvature, mean curvature, the equation of Gauss and Codazzi.

Recommended Books and References:

1. Q. Khan, Differential Geometry of Manifolds, Prentice Hall India 2012, ISBN-9788120346505
2. Jeffry M . Lee, Manifolds and Differential Geometry, Orient Blackswan, 2012, ISBN-9780821887134
3. U C De & A AShaik, Differential Geometry of Manifolds, Narosa Publishing, 2009, ISBN-9788173197772
4. T J Wilmore, An introduction to Differential Geometry, Oxford University Press, 32nd impression, 2017, ISBN-13:9780195611106

COMMUTATIVE ALGEBRA

Theory Credit: 5
Tutorial Credit: 1

UNIT I Preliminaries on rings and ideals; local and semilocal rings; nilradical and Jacobson radical; operations on ideals; extension and contraction ideals; modules and module homomorphisms; submodules and quotient modules; operations on submodules; annihilator of a module; generators for a module, finitely generated modules; Nakayama's lemma; exact sequences.

UNIT II Existence and uniqueness of tensor product of two modules; tensor product of n modules; restriction and extension of scalars; exactness properties of tensor products; flat modules.

UNIT III Multiplicatively closed subsets; saturated subsets; ring of fractions of a ring; localization of a ring; module of fractions and its properties; extended and contracted ideals in a ring of fractions; total ring of fractions of a ring.

UNIT IV Primary ideals; *p*-primary ideals; primary decomposition, minimal primary decomposition, uniqueness theorems; primary submodules of a module.

UNIT V Chain conditions, ascending chain conditions on modules; maximal condition; Noetherian modules; descending chain condition; minimal condition; Artinian modules, their properties; Noetherian rings; Hilbert basis theorem; Artinian rings; structure theorem for Artinian rings.

Recommended Books and References:

1. M. F. Atiyah and I. G. Macdonald, Introduction to Commutative Algebra, Addison Wesley, 2000.
2. M. Reid, Undergraduate Commutative Algebra, London Math. Soc. Student Texts, No. 29, 1995.
3. I. S. Luther and I. B. S. Passi, Algebra (Volume 2: Rings), Narosa Publishing House, New Delhi, 1999.

4. I. S. Luther and I. B . S. Passi, Algebra (Volume 3: Modules), Narosa Publishing House, New Delhi, 1999.
5. S. Lang, Algebra, Addison-Wesley Publishing Company, London, 2000.
6. Gopalakrishnan, Commutative Algebra, Orient BlackswanPvt. Ltd. New Delhi

DISCRETE MATHEMATICS

Theory Credit: 5
Tutorial Credit: 1

UNIT I **Introduction:** Sets, Algebra of sets, Representation of Relation on finite sets, Mapping, composition of mapping, countability of sets, relation, product set, equivalence relation, Principle of Mathematical induction

UNIT II **Logic:** Introduction, proposition and compound properties, basic logical operations, truth tables, tautologies and contradiction, algebra of proposition, conditional and biconditional statement, negation of compound statements, Normal form, negation of quantified statement

UNIT III **Boolean Algebra:** Introduction, Boolean Polynomial, Sum of products and product of sum form, Normal forms, minimal form of Boolean polynomial, prime implicant, Karnaugh Map and Quinn McCluskey method for simplification of Boolean expression, switching circuit and its applications

UNIT IV **Poset and Lattices:** Definition, Examples and basic properties of ordered sets, Hasse diagram, Isomorphic Ordered set, Lattice as Algebraic structure, sub lattices, lattice isomorphism, definition, examples and properties of modular and distributive lattices

UNIT V **Combinatorics:** Principles of addition multiplication, Permutation, combinations, Pigeonhole principle, binomial theorem, multinomial coefficient, Recurrence relation, solution of recurrence relation, homogeneous and non-homogeneous recurrence relation, generating function, solution of recurrence relation by method of generating functions

Recommended Books and References:

1. J P Tremblay and R P Manohar, Discrete Mathematics with Applications to Computer Science, McGraw Hill, 1989, ISBN-978-0074631133
2. C L Liu, Elements of Discrete Mathematics, Tata McGraw Hill, 2005, 4th Ed, ISBN-978-1259006395
3. V K Balakrishnan, Introductory Discrete Mathematics,ISBN-978-0486691152
4. B A Davey and H A Priestley, Introduction to Lattics and order, Cambridge Universtiy Press, Cambridge, 1990, ISBN-9780521134514
5. Rudolf Lidl and Gunter Pilz, Applied Abstract Algebra, Undergraduate text in mathematics, Springer, 2004, 3rd Ed, ISBN-9781441931177
6. Edgar G Goodaire and Micheal M Parmenter, Discrete Mathematics with Graph Theory, Pearson Education, 2003, 3rd Ed, ISBN-978-9332549777
7. S Lipschutz and M L Lipson, Schaum's Outline of Theory and Problems of Discrete Mathematics, 2nd Ed., Tata McGraw-Hill, 1999, ISBN-978-0071615877

LIE ALGEBRA

Theory Credit: 5
Tutorial Credit: 1

UNIT I Generalities.Basic definitions and examples.Structure constants.Relations with Lie groups.Elementary algebraic concepts.Representations; the Killing form.Solvable and nilpotent.Engel's theorem.Lie's theorem.Cartan's first criterion.Cartan's second criterion.Representations of A1. Complete reduction for A1

UNIT II Cartansubalgebra.Roots.Roots for semisimple g. Strings.Cartan integers.Root systems.Weyl group.Root systems of rank two.Weyl-chevalley normal form, first stage.Weyl-chevalley normal form.

UNIT III Compact form. Properties of root systems.Fundamental systems.Classification of fundamental systems.The simple Lie algebras.Automorphisms.

UNIT IV The Cartan-Stiefel diagram. Weights and weight vectors.Uniqueness and existence. Complete reduction.

UNIT V Cartansemigroup; representation ring.The simple Lie algebras.The Weyl character formula.Some consequences of the character formula.Examples.The character ring.Orthogonal and symplectic representations.

Recommended Books and References:

1. Karin Erdmann and Mark J Wildon, Introduction to Lie Algebras, Springer.
2. Brian C Hall, Lie Groups, Lie Algebras and representations- An Elementary Introduction, Springer
3. James E Humphreys, Introduction to Lie Algebras and Representation Theory, Springer.
4. Nathan Jacobson, Lie Algebras, Dover Publications.

THEORY OF RELATIVITY

Theory Credit: 5
Tutorial Credit: 1

UNIT I The special theory of relativity: inertial frames of reference; postulates of the special theory of relativity; Lorentz transformations; length contraction; time dilation; variation of mass; composition of velocities; relativistic mechanics; world events, world regions and light cone; Minkowski space-time; equivalence of mass and energy.

UNIT II Energy-momentum tensors: the action principle; the electromagnetic theory; energy-momentum tensors (general); energy-momentum tensors (special cases); conservation laws.

UNIT III General Theory of Relativity: introduction; principle of covariance; principle of equivalence; derivation of Einstein's equation; Newtonian approximation of Einstein's equations.

UNIT IV Solution of Einstein's equation and tests of general relativity: Schwarzschild solution; particle and photon orbits in Schwarzschild space-time; gravitational red shift; planetary motion; bending of light; radar echo delay.

UNIT V Brans-Dicke theory: scalar tensor theory and higher derivative gravity; Kaluza-Klein theory.

Recommended Books and References:

1. R.K. Pathria, The Theory of Relativity (2nd edition), Hindustan Publishing co. Delhi, 1994.
2. J.V. Narlikar, General Relativity & Cosmology (2nd edition), Macmillan co. of India Limited, 1988.
3. S. K.Srivastava and K. P. Sinha, Aspects of Gravitational Interactions, Nova Science Publishers Inc. Commack, New York, 1998.
4. W. Rindler, Essential Relativity, Springer-Verlag, 1977.
5. R.M. Wald, General Relativity, University of Chicago Press, 1984.

GAME THEORY

Theory Credit: 5
Tutorial Credit: 1

UNIT I **Game Theory Fundamentals:** Historical background. Zero sum games; Non-zero sum games; Extensive Form Games ; Cooperative Games; Bargaining Games; Cooperative versus Non-Cooperative Games.

UNIT II **Two-Person Zero-Sum Games:** Saddle point; Minimax and MaximinStategies; Solving 2xn and mx2 Games; Dominance; Mixed strategy; Linear Programming Methods to solve a Two- person Zero Sum Game.

UNIT III **Two-person Non-Zero-sum Games:** Basic Definitions; Nash equilibrium; Pure and mixed strategies in Nash equilibrium.

UNIT IV **Extensive Form Games:** The Extensive Form; The Strategic Form; Backward induction and subgame perfection; Perfect Bayesian equilibrium.

UNIT V **Cooperative Game Theory:** Cooperative Games with Transferable Utility; The Core; The Shapley value.

Recommended Books and References:

1. Y.Narahari, Game Theory and Mechanism Design, World Scientific,2014
2. S.R.Chakravarty, M. Mitra, P.Sarkar, A Course on Cooperative Game Theory, Cambridge University Press,2015
3. Hans Peter, Game Theory- A Multi-leveled Approach, Springer,2008.

DISSERTATION/PROJECT

Credits: 6