November 2025
M.Sc.
First Semester
CORE – 03
PHYSICS
Course Code: MPHC 1.31
(Mathematical Physics)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five *questions, taking* one *from each unit.*

UNIT–I

1. (a) State and prove Fourier integral theorem. 5

   (b) Find the Fourier sine integral of $f(x) = e^{-\beta x}$, $\beta > 0$ and hence show that $\frac{\pi}{2} e^{-\beta x} = \int_0^\infty \frac{\lambda \sin \lambda x}{\beta^2 + \lambda^2} d\lambda$. 4

   (c) Find the Fourier cosine transform of

   $$f(x) = \begin{cases} x & \text{for } 0 < x < 1 \\ 2 - x & \text{for } 1 < x < 2 \\ 0 & \text{for } x > 2 \end{cases}$$ 5

2. (a) Using Parseval's identity, show that $\int_0^\infty \frac{x^2}{\left(x^2+1\right)^2} dx = \frac{\pi}{4}$.

   Given, $f(x) = \frac{x}{x^2+1}$, $F(s) = \frac{\pi}{2} e^{-s}$. 4

   (b) Find the solution of one-dimensional wave equation using Fourier transform. 10

UNIT–II

3. (a) Evaluate the Laplace transform of $\int_0^t te^{-2t} \sin t \, dt$ using integral theorem. 4

(b) Find the Laplace transform of $F(t)=\begin{cases}\frac{t}{k} & \text{for } 0<t<k\\ 1 & \text{for } t>k\end{cases}$ 5

(c) Obtain the Laplace transform of $t^2e^t\sin 4t$. 5

4. (a) Find the Laplace transform of the following periodic function (Sawtooth wave). 5

$f(t)=\frac{kt}{T}$ for $0<t<T, f(t+T)=f(t)$

(b) Evaluate: $L^{-1}\left[\frac{3s+2}{2s^2-4s+3}\right]$ 4

(c) Using Laplace transform, solve the differential equation: 5

$y''-2y'+2y=0;\ y(0)=y'(0)=1$

UNIT–III

5. (a) What is tensor? 2

(b) Prove that Kronecker delta function is a substitution operator. 3

(c) If $a_{\alpha\beta}x^{\alpha}x^{\beta}=0$ for all values of $x^1,x^2,\ldots x^n$, then show that $a_{ij}+a_{ji}=0$. 4

(d) A covariant tensor of 1st order has components $xy, 2y-z^2, xz$ in rectangular coordinates. Determine its covariant components in spherical polar coordinates. 5

6. (a) Show that the sum of two tensors of same rank results in third tensor of same rank. 3

(b) Prove that skew symmetric tensor of rank 2 has $\frac{N(N-1)}{2}$ independent components in *N* dimensional space. 3

(c) Show that $\frac{\delta g^m k}{\delta x^l}=-g^{jm}\Gamma^k_{jl}-g^{ik}\Gamma^m_{il}$ 4

(d) If $(ds)^2=r^2(d\theta)^2+r^2\sin^2\theta(d\varphi)^2$, find the values of $[22,1]$ and $[1,22]$. 4

UNIT–IV

7. (a) Express $f(x)=4x^3+6x^2+7x+2$ in terms of Legendre polynomial. 4

   (b) Derive modified Bessel differential equation. 5

   (c) Prove the relation $J_0(x)=\frac{1}{\pi}\int_0^{\pi}\cos(x\sin\theta)d\theta$ 5

8. (a) Using Bessel function, derive the relation $xJ_n'(x)=nJ_n(x)-xJ_{n+1}(x)$ 4

   (b) Write down the Hermite differential equation and state the form of its solution. 7

   (c) From the Laguerre differential equation, show that $L_n'(0)=-n$. 3

UNIT–V

9. (a) Explain axis of symmetry. Express axis of symmetry (C_2^z) in matrix form with respect to z axis and hence find its character value. 4

   (b) Workout the reducible representation of ammonia (NH_3) and resolve into irreducible representation. 5

   (c) Find the missing (?) irreducible representation of the following point group: (Given, order = 4.) 5

	E	**X_1**	**X_2**	**X_3**
Γ_1	1	1	1	1
Γ_2	1	–1	1	–1
Γ_3	1	–1	–1	1
Γ_4	?	?	?	?

10. (a) If *a* and *b* are any two elements of a group *G*, then show that the equations $ax=b$ and $ya=b$ have unique solutions in *G*. 4

(b) Let Z be the set of all integers and $*$ be the operation on Z defined $a*b=a+b+1, \forall a,b \in G$. Prove that $(Z,*)$ is an abelian group. 5

(c) Find the generators of the multiplicative cyclic group $(a, a^2, a^3, a^4, a^5, a^6, a^7, a^8 = e)$ of order 8. 5
