November 2025
M.Sc.
First Semester
CORE – 02
PHYSICS
Course Code: MPHC 1.21
(Quantum Mechanics - I)

Total Mark: 70   *Pass Mark: 28*
Time: 3 hours

Answer <u>five</u> questions, taking <u>one</u> from each unit.

UNIT–I

1. (a) What is a Gaussian wave packet? Express its wave functions.   2
    (b) Consider the wave function of a particle

$$\psi(x) = A\left(1 - \frac{x}{a}\right), \text{ for } \frac{a}{2} < x < a.$$

   Find the normalization constant A and also obtain $\langle x \rangle$.   5

    (c) Obtain the energy eigenvalue equation for 1-D linear harmonic oscillator using operator method.   7

2. (a) Prove that the average motion of a wave packet follows Newton's laws of classical mechanics.   8
    (b) Find the probability current density if the wave function is
    $\psi(x) = Ae^{ikx}$.   2
    (c) Give the postulates of quantum mechanics.   4

UNIT–II

3. (a) Show that reflection is inversely proportional to energy E, while transmission is directly proportional to energy E for scattering states of Dirac delta potential.   8
    (b) Write a short note on Hilbert space and Dirac notation.   6

4. (a) Define Hermitian operator. Show that the energy operator is a Hermitian operator.   7
    (b) Obtain the transcendental equation for a finite square well.   7

UNIT–III

5. (a) State and prove viral theorem. 7
   (b) What are spherical harmonics? Find the spherical harmonies for the ground state and first excited state of the hydrogen atom. 7

6. (a) Show that the Jacobi identity
$$\left[\hat{A},\left[\hat{B},\hat{C}\right]\right]+\left[\hat{B},\left[\hat{C},\hat{A}\right]\right]+\left[\hat{C},\left[\hat{A},\hat{B}\right]\right]=0.$$ 5
   (b) Explain degeneracy. Show that the hydrogen atom has n^2 fold degeneracy for n^{th} state. 7
   (c) Why must ϕ be single valued? How does this lead to the restriction that m_l must be an integer? 2

UNIT–IV

7. (a) Describe a rigid rotator and derive the eigenvalue equation for a rigid rotator. 8
   (b) What is Larmor precession? Obtain an expression for Larmor frequency. 6

8. (a) Explain Pauli spin matrices. Give its properties. 6
   (b) Obtain the Clebsch-Gordan coefficient. 8

UNIT–V

9. (a) What are bosons and fermions? Obtain the exchange forces expression for bosons and fermions. 7
   (b) Show that for a symmetric wave function, two particles can simultaneously exist in the same quantum state, while for anti-symmetric wave function; two particles cannot be in the same quantum state. 7

10. (a) Obtain the mathematical formulation of Slater determinant. Find the overall wave function of ground state and first excited state for helium atom using Slater determinant. 8
    (b) Derive the Schrödinger time independent equation for two particle system. 6