November 2025
M.Sc.
Third Semester
DISCIPLINE SPECIFIC ELECTIVE – 02
MATHEMATICS
Course Code: MMAD 3.21
(Graph Theory)

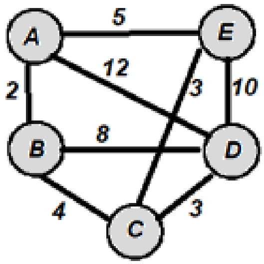
Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five *questions, taking* one *from each unit.*

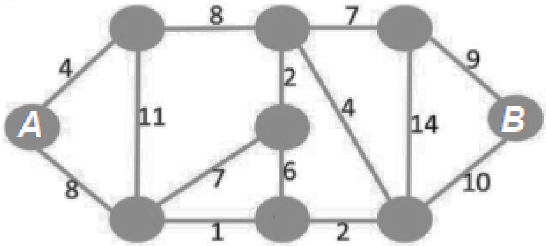
UNIT–I

1. (a) Define with one example each: 1×4=4
(i) Simple graph (ii) Pseudo graph
(iii) Pendant vertex (iv) Isolated vertex
(b) State and prove handshaking theorem. 1+2=3
(c) If a connected or disconnected graph has exactly two vertices of odd degree then there must be a path joining these two vertices. 4
(d) Define Hamiltonian circuit and Hamiltonian path. Give an example for each. Also, draw a graph that has a Hamiltonian path but not a Hamiltonian circuit. 2+1=3

2. (a) Show that the following statement are equivalent for a connected graph G: 5
(i) G is eulerian
(ii) Every vertex is of even degree
(iii) The set of edges of G be partitioned into cycles
(b) State and prove Ore's theorem. 1+4=5
(c) Solve the travelling salesman problem: 4



3. (a) Obtain the shortest distance and the shortest path from vertex A to vertex B in the graph given below: 6



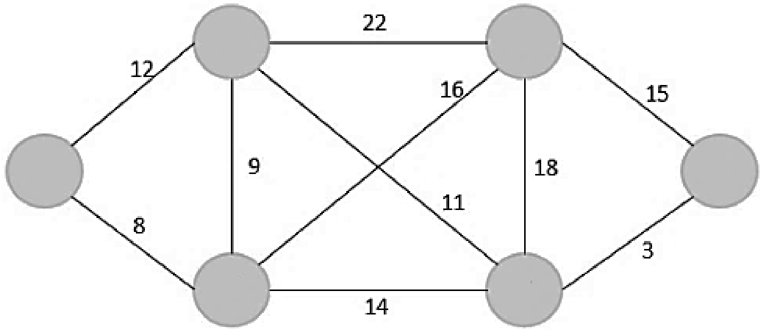
(b) Prove that a graph is a tree of n vertices if and only if it has $n-1$ edges. 4

(c) Is it possible to draw a tree with five vertices having degree 1, 1, 2, 2, 4? 4

4. (a) Let G be a tree with two or more vertices then show that there exist at least two pendant vertices. 4

(b) Prove that radius $\leq$ diameter $\leq$ 2 times radius for any connected graph. 5

(c) Find all the minimal spanning tree using Prim's algorithm: 5



UNIT–III

5. (a) In a graph G if S is a cut set, then show that G-S has two components. 5

(b) Let G be a connected graph. Prove that a vertex $v \in V(G)$ is a cut vertex if and only if there exist $u, w \in V(G), u, w \neq v$, such that v is on every uw path of G. 5

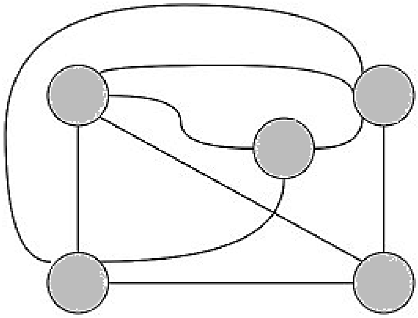
(c) If T is a spanning tree, then show that a branch b_i that determines a fundamental cut set S is contained in every fundamental circuit associated with the chords in S and in no others. 4

6. (a) Show that $K_{3,3}$ is not coplanar. 3

(b) A connected graph has nine vertices having degree 2, 2, 2, 3, 3, 3, 4, 4 and 5. How many edges are there? How many faces are there? 3

(c) Find the thickness and crossing of the graph $K_{3,4}$. 2+2=4

(d) Check the planarity of the given graph: 4



UNIT–IV

7. (a) Define the following: 1×4=4

(i) Chromatic partitioning

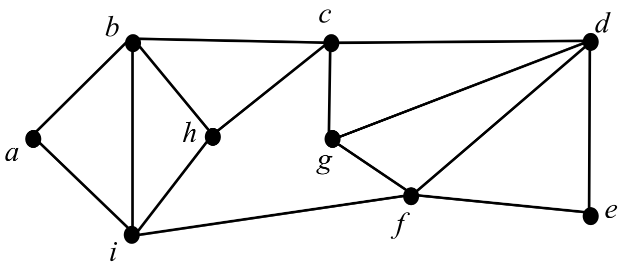
(ii) Uniquely colourable graph

(iii) Dominating set of vertices

(iv) Complete or perfect matching

(b) Prove that a graph with at least one edge is 2-chromatic if and only if it has no circuit of odd length. 4

(c) Find the chromatic polynomial of the graph: 6



8. (a) Prove that the chromatic number is less than the maximum degree of vertices in a graph *G*. 5

(b) Let *G* be a graph of *n* vertices, then show that *G* is a complete graph if and only if its chromatic polynomial $P_n(\lambda)=\lambda(\lambda-1)(\lambda-2)...(\lambda-n+1)$. 7

(c) State the four color theorem. 2

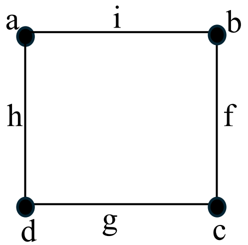
UNIT–V

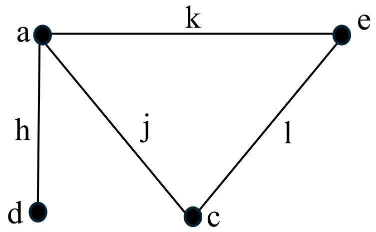
9. (a) Define the following: 1×4=4

(i) Balanced digraph (ii) Circuit subspaces

(iii) Cut set subspaces (iv) Nullity of a graph

(b) Find the union, intersection, and ring sum of the following graphs: 4





(c) Show that the set consisting of all the circuit and the edge-disjoint union of circuit (including the null set ϕ) in a graph is an abelian group under ring sum operation. 6

10. (a) Show that the number of circuit vectors (including the null set ϕ) in W_Γ is 2^μ where μ is the nullity of the graph. 4

(b) Prove that the intersection of the circuit subspace and cut set subspace of *G* is a subspace of the graph *G*. 5

(c) Show that the set of cut set vectors corresponding to the set of fundamental cut sets w.r.t. any spanning tree forms a basis for the cut set subspaces W_S. 5