

November 2025
M.Sc.
Third Semester
CORE – 09
MATHEMATICS
Course Code: MMAC 3.11
(Partial Differential Equation)

Total Mark: 70

Pass Mark: 28

Time: 3 hours

Answer five questions, taking one from each unit.

NB: Symbols have their usual meaning.

UNIT-I

1. (a) Find the general integral of $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$. 5
- (b) Find the integral surface of the linear PDE $(x - y)p - (x - y + z)q = z$ which contains $z = 1$ and $x^2 + y^2 = 1$. 5
- (c) Solve: $9(p^2z + q^2) = 4$ 4
2. (a) Find the complete integral of $x^2p^2 + y^2q^2 - 4 = 0$ by Charpit's method. 5
- (b) Find the surface that is orthogonal to the system $z = cxy(x^2 + y^2)$ and passes through the hyperbola $x^2 - y^2 = a^2, z = 0$. 5
- (c) Solve: $p^2 + q^2 = 1$ 4

UNIT-II

3. (a) Reduce the equation $u_{xx} + 2xu_{xy} + x^2u_{yy} = 0$ to canonical form. 5
- (b) Solve $3r + 4s + t + (rt - s^2) = 1$ by Monge's method. 5

(c) Construct the adjoint operator $L^*(v)$ of the equation

$$L(u) = a(x)u_{xx} + b(x)u_{xy} + c(x)u_{yy}. \quad 4$$

4. (a) Solve the following: 5×2=10

(i) $(D^2 - DD')u = \cos x \cos 2y$

(ii) $(x^2D^2 + 2xyDD' + y^2D'^2)u = x^2y^2$

(b) Solve: $xyr + x^2s - yp = x^3e^y$ 4

UNIT-III

5. (a) Define a harmonic function. State and prove the mean value theorem for a harmonic function. 1+6=7

(b) Show that in cylindrical coordinates (r, θ, z) the Laplace

equation $\nabla^2 u = 0$ takes the $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2} = 0$. 7

6. (a) Find the solution of the interior Dirichlet problem for a circle. 7

(b) Find the steady state temperature distribution in a semi-circular plate of radius a insulated on both the faces with its curved boundary kept at a constant temperature U_0 and its bounding diameter kept at zero temperature. 7

UNIT-IV

7. (a) Derive the diffusion equation across a surface. 7

(b) Solve the following initial boundary value problem defined by

$$\left. \begin{aligned} u_t &= u_{xx}, 0 < x < 3, t > 0 \\ u(0, t) &= 0, u(3, t) = 3 \\ u(x, 0) &= 4x - x^2 \end{aligned} \right\} \quad 7$$

8. (a) Obtain the solution of diffusion equation in cylindrical coordinates system. 7

(b) A bar 10 cm long with insulated sides, has its ends A and B kept at 20°C and 40°C respectively until steady state conditions prevail. Temperature at A is then suddenly raised to 50°C and at

the same instant that at B is lowered to 10°C . Find the subsequent temperature at any point on the bar at any time. 7

UNIT-V

9. (a) Solve the following non-homogeneous wave equation defined by:
$$\left. \begin{aligned} u_{tt} - c^2 u_{xx} &= f(x, t), -\infty < x < \infty, t > 0 \\ u(x, 0) &= \phi(x), u_t(x, 0) = \psi(x) \end{aligned} \right\} 7$$

- (b) Make use of the method of separation of variables to solve the following initial boundary value problem defined by
$$\left. \begin{aligned} u_{tt} &= c^2 u_{xx}, 0 \leq x \leq L, t > 0 \\ u(0, t) &= 0, u(L, t) = 0, t > 0 \\ u(x, 0) &= f(x), u_t(x, 0) = g(x) \end{aligned} \right\} 7$$

10. (a) Obtain the D'Alembert's solution of the
$$u_{tt} = c^2 u_{xx}, -\infty < x < \infty, t > 0$$
 subject to initial conditions:
$$u(x, 0) = f(x), u_t(x, 0) = g(x).$$
 Also, discuss its physical significances. 5+2=7
- (b) Obtain the periodic solution of one-dimensional wave equation in spherical polar coordinates. 7