**October 2025**
**B.A./B.Sc.**
**Third Semester**
MINOR – 3
**STATISTICS**
*Course Code: STN 3.11*
(Statistical Inference)

*Total Mark: 50* *Pass Mark: 20*
*Time: 2 hours*

*I. Answer* three *questions, taking* one *from each unit.*

## UNIT–I

1. (a) What is unbiasedness? Show that $t = \frac{1}{n}\sum_{i=1}^{n} x_i^2$ is an unbiased estimator of $\mu^2 + 1$, if $X \sim N(\mu, 1)$. 2+4=6

   (b) Estimate $\alpha$ and $\beta$ by the method of moments for the distribution $f(x;\alpha,\beta) = \frac{\beta^\alpha}{\Gamma\alpha} x^{\alpha-1} e^{-\beta x}, 0 \le x < \infty$. 6

2. (a) A random sample $(X_1, X_2, X_3, X_4, X_5)$ of size 5 is drawn from a normal population with unknown mean. Consider the following estimators to estimate $\mu$:

   (i) $t_1 = \frac{X_1 + X_2 + X_3 + X_4 + X_5}{5}$

   (ii) $t_2 = \frac{X_1 + X_2}{2} + X_3$

   (iii) $t_3 = \frac{2X_1 + X_2 + \lambda X_3}{3}$ where $\lambda$ is such that $t_3$ is an unbiased estimator of $\mu$. Find $\lambda$. Are $t_1$ and $t_2$ unbiased? State giving reasons, the estimator which is best among $t_1, t_2$ and $t_3$. 7

   (b) Explain the concept of Bayes' estimators and give some of its properties. 5

## UNIT–II

3. (a) State and prove weak law of large numbers (WLLN). 6

(b) Obtain the MVB estimator for μ in normal population $N(\mu, \sigma^2)$, where $\sigma^2$ is known. 6

4. (a) What are confidence intervals? Given one observation from a population with p.d.f. $f(x,\theta) = \frac{2}{\theta^2}(\theta - x)$, $0 \le x \le \theta$. Obtain $100(1-\alpha)\%$ confidence interval for θ. 2+5=7

(b) Define central limit theorem (CLT) and write its application. 2+3=5

## UNIT–III

5. (a) What do you understand by sample variance and sample proportion? 2

(b) If $x \ge 1$ is the critical region for testing $H_o : \theta = 2$ against $H_1 : \theta = 1$, on the basis of the single observation from the population, $f(x,\theta) = \theta \exp(-\theta x)$; $0 \le x < \infty$. Obtain the values of type-I and type-II errors. 6

(c) Define simple and composite statistical hypothesis. How is a statistical hypothesis tested? 4

6. (a) State Neyman-Pearson lemma. Let $p$ be the probability that a coin will fall head in a single toss in order to test $H_0 : p = \frac{1}{2}$ against $H_1 : p = \frac{3}{4}$. The coin is tossed 5 times and $H_0$ is rejected if more than 3 heads are obtained. Find the probability of type-I error and power of test. 2+6=8

(b) Define likelihood ratio test and state its important properties. 2+2=4

*II. Answer* any two *of the following questions.*

7. In random sampling from normal population $N(\mu,\sigma^2)$ with p.d.f.

$$f(x,\mu,\sigma^2)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left\{-\frac{1}{2\sigma^2}(x_i-\mu)^2\right\};\ -\infty<x<\infty,$$

$-\infty<\mu<x$, $\sigma>0$, find the MLE for 7

(a) $\mu$ when $\sigma^2$ is known

(b) $\sigma^2$ when $\mu$ is known

8. State and prove Lindeberg-Levy theorem, clearly mentioning the assumptions made. 7

9. Define the following with usual notations: 2+2+2+1=7
   (a) Most powerful test
   (b) Type-I and type-II errors
   (c) Uniformly most powerful test
   (d) Random sample