October 2025
B.A./B.Sc.
Third Semester
MAJOR – 4
STATISTICS
Course Code: STM 3.21
(Calculus)

Total Mark: 75 *Pass Mark: 30*
Time: 3 hours

Answer <u>any five</u> *questions, taking* <u>at least one</u> *from each unit.*

UNIT–I

1. (a) State and prove Euler's theorem on homogeneous function. 1+4=5

(b) Solve the given equations using L-Hospital's rule: 2+2=4

(i) $\lim_{x\to 2}\frac{x^6-24x-16}{x^3+2x-12}$

(ii) $\lim_{x\to\infty}\frac{x+\cos x}{x}$

(c) Define limit of a function. If $f(x)=\frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}$, does the limit exist at $x=0$? 1+3=4

(d) If $f(x)=\frac{x^2-1}{x-1}$, find the limit of $f(x)$ as x tends to 1. 2

2. (a) A function $f(x)$ is defined by,

$$f(x)=\begin{cases} -x^2, \text{if } x<0 \\ 5x-4, \text{if } 0<x\le 1 \\ 4x^2-3x, \text{if } 1<x<2 \\ 3x+4, \text{if } x\ge 2 \end{cases}$$

Examine $f(x)$ for continuity at $x = 0,1,2$. Also discuss the kind of discontinuity. 5

(b) State and prove Leibnitz rule. 5

(c) Define differentiability of a function. Check whether the function $f(x) = |x = 5|$ is differentiable at $x = 5$. 2+2=4

(d) Evaluate $\lim_{x \to 0} \frac{3x^2 + 4x}{6x^2 + 7x}$. 1

UNIT–II

3. (a) Derive the theorem for transformation of variable using Jacobian of transformation. 4

(b) Define maxima of a function. Show that $x^5 - 5x^4 + 5x^3 - 1$ is maximum when $x = 1$ and minimum when $x = 3$. 1+3=4

(c) Change the order of integration and evaluate the following,

$$\int_0^a \int_y^a \frac{x}{x^2 + y^2} dxdy$$ 3

(d) Evaluate the following integration

(i) $\int_1^2 \int_1^2 \frac{1}{xy} dxdy$

(ii) $\iint_R x^2 + y^2 dxdy$; where R is the bond/plane region.

$1 \le x \le 0 ; 1 \le y \le 0$ 2+2=4

4. (a) Write the necessary and sufficient condition for determining maxima and minima of one variable. 4

(b) If $x = r\sin\theta\cos\phi, y = r\sin\theta\cos\phi, z = r\cos\theta$. Find

$$J = \frac{\delta(X, Y, Z)}{\delta(r, \theta, \phi)}$$ 4

(c) Define differential under integral sign. Prove that

$$\int_0^1 \frac{x^\alpha - 1}{\log x} dx = \log(1+\alpha)$$ 4

(d) Derive Lagrange multiplier theorem and construct the required equation. 3

UNIT–III

5. (a) Prove that the necessary and sufficient condition for the ordinary differential equation $Mdx+Ndy=0$ to be exact is that $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$. 5

(b) Solve $\left(xy\sin xy + \cos xy\right) ydx + \left(xy\sin xy - \cos xy\right) xdy$ 4

(c) Solve $x^2(y - px) = yp$ by Clairaut's equation. 3

(d) Solve $(D^2 - 5D + 6)y = sin3x$ 3

6. (a) Write down the equation solvable for p and solve

$$\frac{dy}{dx}\left(\frac{dy}{dx} + y\right) = x\left(x + y\right)$$ 2+3=5

(b) Solve $y\sin 2xdx - (y^2 + \cos^2 x)dy = 0$ by exact differential equation. 4

(c) Solve: $\frac{d^3y}{dx^3} - 9\frac{d^2y}{dx^2} + 23\frac{dy}{dx} - 15y = 0$ 3

(d) Solve: $(D^3 + 6D^2 + 11D + 6)Y = e^{2x}$ 3

UNIT–IV

7. (a) Form the partial differential equation by eliminating the arbitrary constant *a, b* from $z = ax + by + cxy$. 4

(b) Solve the nonlinear equation $\sqrt{p} + \sqrt{q} = 1$. 4

(c) Write down the general solution and the working method for solving $Pp + Qq = R$ by Lagrange's method. 4+3=7

8. (a) Form the partial differential equation by eliminating the arbitrary function f and ϕ from $z = yf(x) + x\phi(y)$. 4

(b) Find the complete integral of $z^2\left(p^2q^2 + q^2\right) = 1$. 4

(c) Solve Charpit's equation by $pxy + pq + qy = yz$. 4

(d) Solve the non-linear equation $pq = xy$. 3