

October 2025
B.A./B.Sc.
Third Semester
MAJOR – 3
STATISTICS
Course Code: STM 3.11
(Statistical Inference)

Total Mark: 50
Time: 2 hours

Pass Mark: 20

I. Answer three questions, taking one from each unit.

UNIT-I

1. (a) Define estimator and estimate of the parameter θ with one example each. 6

(b) For the double Poisson distribution

$$p(x) = P(X = x) = \frac{1}{2} \frac{e^{-m_1} m_1^x}{x!} + \frac{e^{-m_2} m_2^x}{x!}, \quad x = 0, 1, 2, \dots$$

Show that the estimates from m_1 and m_2 by the method of moments are $\mu'_1 \pm \sqrt{\mu'_2 - \mu'_1 - \mu_1'^2}$ 6

2. (a) Explain the method of minimum chi-square of estimation. 4

(b) X_1, X_2 and X_3 is a random sample of size 3 from a population with mean value μ and variance σ^2 . T_1, T_2, T_3 are the estimators used to estimate mean value μ , where

$$T_1 = X_1 + X_2 - X_3; \quad T_2 = 2X_1 + 3X_3 - 4X_2 \quad \text{and}$$

$$T_3 = \frac{1}{3}(\lambda X_1 + X_2 + X_3)$$

(i) Are T_1 and T_2 unbiased estimators? 2

(ii) Find the value of λ such that T_3 is unbiased estimator for μ . 2

(iii) With this value of λ , is T_3 a consistent estimator? 1

(iv) Which is the best estimator? 3

UNIT-II

3. (a) State and prove De-Moivre Laplace theorem. 6
(b) For geometric distribution $f(x) = 2^{-x}; x = 1, 2, 4, \dots$ prove that Chebychev's inequality gives $P\{|X - 2| \leq 2\} > \frac{1}{2}$, while the actual probability is $\frac{15}{16}$. 6
4. (a) Prove that a necessary and sufficient condition for sequence $\{X_n\}$ to satisfy the W.L.L.N. is $E\left\{\frac{Y_n^2}{1+Y_n^2}\right\} \rightarrow 0$ as $n \rightarrow \infty$, where $\{X_n\}$ is any sequence of r.v.'s and given $Y_n = \frac{\{S_n - E(S_n)\}}{n}$ and $S_n = X_1 + X_2 + \dots + X_n$. 5
(b) Define convergence in probability. 2
(c) Obtain the MVB estimator for μ in normal population (μ, σ^2) , where σ^2 is known. 5

UNIT-III

5. (a) Describe likelihood ratio test and state its important properties. 4+2=6
(b) Given the frequency function:
$$f(x, \theta) = \begin{cases} \frac{1}{\theta}, & 0 \leq x \leq \theta \\ 0, & \text{elsewhere} \end{cases}$$
and that you are testing the null hypothesis $H_0 : \theta = 1$ against $H_1 : \theta = 2$, by means of a single observed value of x . What would be the sizes of the Type-I and Type-II errors, if you choose the interval $0.5 \leq x$ and $1 \leq x \leq 2$ as the critical regions? Also, obtain the power function of the test. 6
6. (a) Explain critical region and level of significance. 4
(b) Define sampling distribution and standard error. 2

- (c) Let p be the probability that a coin will fall head in a single toss in order to test $H_0 : p = 0.5$ against $H_1 : p = 0.75$. The coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtained. Find the probability of Type-I error and power of test.

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II. Answer any two of the following questions.

7. (a) Find the M.L.E. for the parameter λ of a Poisson distribution on the basis of a sample size n . 3
 (b) What is Consistency? If X_1, X_2, \dots, X_n are random observations on a Bernoulli variate X taking the value 1 with probability p and the value 0 with probability $(1-p)$, show that

$$\frac{\sum x_i}{n} \left(1 - \frac{\sum x_i}{n} \right) \text{ is a consistent estimator of } p(1-p). \quad 1+3=4$$

8. State and prove Rao-Blackwell theorem. 7
 9. Let X follows normal distribution with p.d.f.

$$f(x, \theta) = \frac{1}{\sigma\sqrt{2\pi}} \exp \left\{ -\frac{1}{2} \left(\frac{x - \theta}{\sigma} \right)^2 \right\}, -\infty < x < \infty$$

where σ is known. For testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1 (> \theta_0)$ construct S.P.R.T. and obtain its OC function. 7