October 2025
B.A./B.Sc.
Fifth Semester
CORE – 11
STATISTICS
Course Code: STC 5.11
(Stochastic Processes & Queuing Theory)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five *questions, taking* one *from each unit.*

UNIT–I

1. (a) Define probability generating function (p.g.f.). Can $P(s)=\frac{2}{1+s}$ be the probability generating function of a random variable *X*? Give reason. 2+4=6

   (b) Let *Y* have the distribution of geometric form given by,
   $$P_r(Y=k)=g^{k-1}.p;\ k=1,2,3,...$$
   Show that the p.g.f. of *Y* is $\frac{ps}{1-qs}$ and $E(Y)=\frac{1}{p}$ and $Var(Y)=\frac{q}{p^2}$. 5

   (c) If $P(s)$ is the p.g.f. of a random variable *X,* find the p.g.f. of $\frac{X-m}{n}$, where *m* and *n* are any arbitrary constants. 3

2. (a) If $P(s)$ is the probability generating function (p.g.f.) of a random variable *X*, then find the p.g.f. of: 2×2=4
   (i) $X+1$ (ii) $2X$

   (b) Let *X* be a random variable with probability mass function $P(x=n)=q^{n-1}p$ for $n=1,2,3,\ldots$. Obtain the probability generating function of *X*, $E(X)$ and $SD(X)$. 6

(c) Write a note on specification of stochastic process. Define hazard function. 3+1=4

UNIT–II

3. (a) Define the following: 2×2=4
   (i) Markov chain
   (ii) Stability of Markov chain.

   (b) Let $\{X_n, n \geq 0\}$ be a Markov chain with three states 0, 1, 2 and with transition probability matrix $\begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{bmatrix}$. The initial probability distribution is $P(X = i) = \frac{1}{3}$; for i = 1, 2, 3. Find

   (i) $P(X_2 = 2, X_1 = 1 \mid X_0 = 2)$ 2×3=6

   (ii) $P(X_2 = 2, X_1 = 1, X_0 = 2)$

   (iii) $P(X_2 = 1 \mid X_0 = 1)$

   (c) For the given Markov chain having transition probability matrix $\begin{bmatrix} 1-m & m \\ n & 1-n \end{bmatrix}$, derive for the generalization of independent Bernoulli trials. 4

4. (a) Define the following: 3×2=6
   (i) Unit-step and m-step transition probability
   (ii) Transition probability matrix

   (b) For a higher transition probability, state and prove Chapman-Kolmogorov equation. 8

UNIT–III

5. (a) Define Poisson process. State and derive any two properties of Poisson process. 2+4+4=10

   (b) Show that if n, the number of arrivals in time t, follows the Poisson distribution, then the time between two consecutive arrivals follows the negative exponential distribution. 4

6. (a) Derive the birth and death process. 10

(b) Obtain the differential-difference equations in case of pure death process. 4

UNIT–IV

7. (a) What is queueing theory? Show that, if the arrivals are completely random, then the probability distribution of number of arrivals in a fixed time-interval follows a Poisson distribution. 2+7=9

(b) Obtain probability density function of waiting time (excluding service time) distribution. 5

8. (a) Define transient and steady states. Derive the distribution of inter-arrival times. 2+6=8

(b) For queueing Model-1, find 6

(i) expected number of units in the system $\left(L_s\right)$

(ii) expected queue length $\left(L_q\right)$.

UNIT–V

9. (a) Explain price elasticity of demand and price elasticity of supply. 6

(b) The demand function of two commodities A and B are $D_A = 10 - p_A - 2p_B$ and $D_A = 10 - p_A - 2p_B$ and the corresponding supply functions are $S_A = -3 + p_A + p_B$ and $S_B = -2 + p_B$ where p_A and p_B denote the prices of A and B respectively. Find the equilibrium process and quantities exchanged in the market. 4

(c) Describe the Pareto's law of income distribution. 4

10. (a) Explain the two types of data required for estimating elasticities. 6

(b) The demand function for a commodity X is given by $x = 300 - 0.5p_x^2 + 0.02p_0 + 0.05y$, where x is the quantity demanded of X, p_x is the price of X, p_0 the price of a related commodity and y is the constant income. Compute 4

(i) the price elasticity of demand for X

(ii) the income elasticity of demand for X, when $p_x = 12$, $p_0 = 10$ and $y = 200$.

(c) Explain Engel's curves for constant prices and income. 4
