October 2025
B.A./B.Sc.
Third Semester
MINOR – 3
PHYSICS
Course Code: PHN 3.11
(Mathematical Physics - I)

Total Mark: 50 *Pass Mark: 20*
Time: 2 hours

I. Answer three *questions, taking* one *from each unit.*

UNIT–I

1. (a) Determine the directional derivative of the scalar field $\phi = xy^2z^3$ in the direction $(\hat{i} - 2\hat{j} + 2\hat{k})$ at a point $(3,1,-1)$. 6

   (b) If $\vec{F} = 3xy\hat{i} + y^2\hat{j}$, evaluate $\int_c \vec{F}.\overrightarrow{dr}$, where c is the curve $y = 2x^2$ from $(0,0)$ to $(1,2)$. 6

2. (a) Verify Green's theorem in the plane for $\int_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$; where c is the boundary of the region defined by $x = 0$, $y = 0$ and $x + y = 1$. 6

   (b) Prove that vector point $\vec{f}$ is solenoidal vector at $(1,0,-1)$, where $\vec{f} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$. 3

   (c) Show that $\vec{\nabla}.(\vec{\nabla} \times \vec{f}) = 0$. 3

UNIT–II

3. (a) Solve the initial value problem $\frac{dy}{dx} = y^2(1 + x)$; with initial value $y(0) = 1$. 6

(b) If $\left(4x^3+6xy+y^2\right)dx+\left(3x^2+2xy+2\right)dy=0$. 6

(i) Is the above equation an exact equation?

(ii) If so, find its solution.

4. (a) Find the Wronskian of the solutions $y_1=\sin 3x$ and $y_2=\cos 3x$. Comment whether the solutions are linearly depend or not. 3

(b) Write the three commonly used partial differential equations in physics. 3

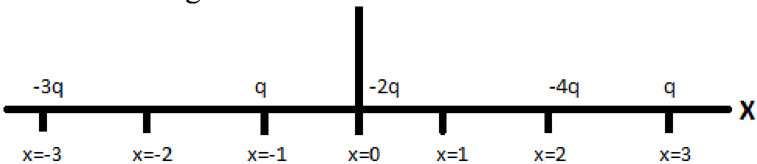
(c) Solve the following equation by successive integration method $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=x$. 6

UNIT–III

5. (a) Evaluate the following integral by using the properties of Dirac's delta function (δ) $\int_1^4 x^4\delta(x-3)dx$ 3

(b) Explain the orthogonal curvilinear coordinate system (OCCS) and find an expression for the gradient of a scalar field ϕ in OCCS. 2+5=7

(c) By using δ function, express the following charge distribution as shown in the figure below



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6. (a) Show the Dirac's delta function (δ) as a limit of Gaussian distribution function. 4

(b) Show that $\delta^2(x)=(-1)^n n!\frac{\delta(x)}{x}$. 3

(c) Derive an expression for curl of a vector function in orthogonal curvilinear coordinate system. 5

II. Answer any two *questions from the following.*

7. (a) Electric filed inside a uniformly charged solid sphere is $\vec{E} = \frac{\rho\vec{r}}{3\varepsilon_0}$, find $div\,\vec{E}$. 3

(b) Show that curl of a central field of $\vec{F} = f(r)\vec{r}$ is a conservative field, here $\vec{r}$ is position vector. 4

8. 1-D heat equation in a rod of length is governed by $\frac{\partial^2\theta}{\partial x^2} = \frac{1}{\sigma}\frac{\partial\theta}{\partial t}$. Solve the above equation subject to boundary condition $\theta(0,t) = 0$ and $\theta(L,t) = 0$. 7

9. Using generalised orthogonal curvilinear coordinate system, find the value of scale factor for cylindrical coordinate system. Also, find the expression for gradient, divergence, and Laplacian operator.
2+1+2+2=7