**October 2025**
**B.A./B.Sc.**
**Third Semester**
MAJOR – 3
**PHYSICS**
*Course Code: PHM 3.11*
(Mathematical Physics - I)

*Total Mark: 50* *Pass Mark: 20*
*Time: 2 hours*

*I. Answer* three *questions, taking* one *from each unit.*

## UNIT–I

1. (a) Solve the differential equation:
$$\left(x^3 - x\right)\frac{dy}{dx} - \left(3x^2 - 1\right) y = x^5 - 2x^3 + x$$ 4

(b) If $y_1 = e^{-x}\cos x$, $y_2 = e^{-x}\sin x$ and $\frac{d^2y}{dx^2} + \frac{2dy}{dx} + 2y = 0$, then 2+1+1=4

(i) calculate the Wronskian determinant.
(ii) verify that $y_1$ and $y_2$ satisfy the given differential equation.
(iii) apply Wronskian test to check that $y_1$ and $y_2$ are linearly independent.

(c) Solve the partial differential equation:
$$x\left(y^2 + z\right)p - y\left(x^2 + z\right)q = z\left(x^2 - y^2\right)$$
where, $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$ 4

2. (a) Obtain the complete solution of the differential equation:
$$\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 2y = e^x + \cos x$$ 4

(b) Solve and obtain the general solution of the differential equation:
$$y'' - 2y' + 2y = x + e^x \cos x$$ 5

(c) Check whether the following equations are exact differentials or not: 3

(a) $\left(x^2+y^2+3\right)dx-2xydy=0$

(b) $\left(y^4+2y\right)dx+\left(xy^3+2y^4-4x\right)dy=0$

## UNIT–II

3. (a) Find the angle between the surfaces $x^2+y^2+z^2=9$ and $z=z^2+y^2-3$ at the point $\left(2,-1,2\right)$. 4

(b) Prove that $(y^2-z^2+3xy-2x)\hat{i}+(3xz+2xy)\hat{j}+(3xy-2xz+2z)\hat{k}$ is both solenoidal and irrotational. 4

(c) If $f$ and $g$ are two scalar point functions, prove that: $div\ \left(f\nabla h\right)=f\nabla^2 g+\nabla f\nabla g$ 4

4. (a) Evaluate $\int\int_S\left(yz\hat{i}+zx\hat{j}+xy\hat{k}\right).\overrightarrow{ds}$, where $S$ is the surface of the sphere $x^2+y^2+z^2-a^2$ in the first octant. 4

(b) If $\vec{F}=2xz\hat{i}-x\hat{j}+y^2\hat{k}$, evaluate $\iiint\vec{F}dV$ over the region bounded by the surfaces $x=0$, $y=0$, $y=6$ and $z=x^2$, $z=4$. 4

(c) Use Divergence theorem to evaluate $\iint_S\vec{A}.\overrightarrow{ds}$, where $\vec{A}=x^3\hat{i}+y^3\hat{j}+z3\hat{k}$, and $S$ is the surface of the sphere $x^2+y^2+z^2=a^2$. 4

## UNIT–III

5. (a) Derive the expression of curl $\overline{f}$ in the orthogonal curvilinear co-ordinates. 4

(b) Show that the spherical co-ordinate system $\left(\overline{T}_r\ ,\overline{T}_\theta,\ \overline{T}_\phi\right)$ is self-reciprocal. 3

(c) Express $z\hat{i}-2x\hat{j}+y\hat{k}$ in cylindrical co-ordinates. 5

6. (a) Define Dirac delta function and show that

$$\delta\left(x^2 - a^2\right) = \frac{\delta(x-a)+\delta(x+a)}{2|a|}$$ 1+4=5

(b) Show that $\int_0^\infty f(t)\delta(t)dt = f(0)$. 3

(c) Show that the Gaussian function, $G_\sigma(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[\frac{-x^2}{2\sigma^2}\right]$,

$\sigma > 0$ has a unit area under its curve. 4

*II. Answer* <u>any two</u> *questions from the following.*

**UNIT–I**

7. Find the complete solution for the differential equation:

$$\frac{d^3y}{dx^3} - 7\frac{d^2y}{dx^2} + 10\frac{dy}{dx} = e^{2x}\sin x.$$ 7

**UNIT–II**

8. Show that $\text{div}\left(\text{grad}\, r^n\right) = n(n+1)r^{n-2}$, where $r = \sqrt{x^2+y^2+z^2}$

Hence, show that $\nabla^2\left(\frac{1}{r}\right) = 0$. 7

**UNIT–III**

9. State Stoke's theorem. Verify Stoke's theorem for $\vec{A} = (2x-y)\hat{i} + yz^2\hat{j} - y^2z\hat{k}$, where $S$ is the upper half surface of the sphere $x^2+y^2+z^2 = 1$ and $c$ is its boundary. 7

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