October 2025
B.A./B.Sc.
First Semester
MINOR – 1
MATHEMATICS
Course Code: MAN 1.11
(Algebra)

Total Mark: 50 *Pass Mark: 20*
Time: 2 hours

I. Answer three *questions, taking* one *from each unit.*

UNIT–I

1. (a) Find the quotient and the remainder $3x^4 - 5x^3 + 10x^2 + 11x - 61$ is divided by $x - 3$. 3
   (b) State and prove the factor theorem. 5
   (c) If α, β, γ are roots of $x^3 - x^2 + 4x - 1 = 0$, evaluate $\alpha^2 + \beta^2 + \gamma^2$. 4

2. (a) Solve the biquadratic equation $x^4 + x^3 - 16x^2 - 4x + 48 = 0$ if product of two of its roots is 6. 5
   (b) Solve the cubic equation $3x^3 - 26x^2 + 52x - 24 = 0$, the roots being in geometrical progression. 4
   (c) Express the symmetric sum $\alpha^3 + \beta^3 + \gamma^3$ in terms of the coefficients of the cubic equation $x^3 + px^2 + qx + r = 0$. 3

UNIT–II

3. (a) Define the following with examples: Hermitian, skew-Hermitian, idempotent, nilpotent. 4
   (b) Verify that the matrix $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$ is involutory. 3

(c) Find the products AB and BA, where $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 0 & 4 \\ 0 & 5 & 1 \end{bmatrix}$,

$B = \begin{bmatrix} -1 & 0 & 3 \\ 2 & 0 & 4 \\ -3 & 1 & 0 \end{bmatrix}$. Are they equal? 5

4. (a) Prove that a square matrix A can be expressed in a unique way as a sum of a symmetric matrix and a skew-symmetric matrix. 5

(b) Find the inverse of the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$. 4

(c) Compute the product $\begin{bmatrix} 1 & 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 4 \\ -1 \end{bmatrix}$. 3

UNIT–III

5. (a) Define trace and rank of a matrix with examples. 3

(b) Reduce the matrix $\begin{bmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & -1 & 2 & 0 \end{bmatrix}$ to echelon form and determine its rank. 4

(c) Using elementary row operations, find the inverse of $A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & -1 & 3 \\ 3 & -1 & -1 \end{bmatrix}$. Hence, using it, solve the system of linear equations $\begin{cases} x + y + 2z = 4 \\ 2x - y + 3z = 9 \\ 3x - y - z = 2 \end{cases}$. 5

6. (a) What is the rank of a square matrix of order three, if every element of the matrix is unity? 3

(b) Reduce the matrix $\begin{bmatrix} 1 & -1 & 2 & -3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 0 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}$ to normal form and determine its rank. 4

(c) Express the matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ as a product of elementary matrices. Hence find A^{-1}. 5

II. Answer <u>any two</u> *questions from the following.*

7. (a) Use the remainder theorem to find the remainder when $x^4 - 2x^3 + 3x^2 - 5x + 7$ is divided by $(x-2)$. 2

(b) If α and β are roots of $x^2 - 5x + 6 = 0$, find $\alpha^2 + \beta^2$ and $\alpha^3 + \beta^3$. 3

(c) Form a quadratic equation whose roots are $\sqrt{2} + \sqrt{3}$ and $\sqrt{2} - \sqrt{3}$. 2

8. (a) Define an involutory matrix with an example. 2

(b) For $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, compute $(A - I_3)^3$. Hence, determine the least positive integer *k* such that $(A - I_3)^k = 0$. (I_3 is the identity matrix of order 3) 2

(c) Prove that every orthogonal matrix is non-singular. 3

9. (a) Under what condition the matrix equation $A^2 - B^2 = (A + B)(A - B)$ true? 3

(b) Find the trace and rank of $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \end{bmatrix}$. 2

(c) Find the minor of the element a_{11} in the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$.

Also, find the trace of the matrix. 2
