October 2025
B.A./B.Sc.
Third Semester
MAJOR – 4
MATHEMATICS
Course Code: MAM 3.21
(Real Analysis - I)

Total Mark: 75 *Pass Mark: 30*
Time: 3 hours

Answer five *questions, taking* at least one *from each unit.*

UNIT–I

1. (a) If $a, b \in \mathbb{R}$, prove the following: 4
   (i) $a.b = 0$, then either $a = 0$ or $b = 0$
   (ii) $a \neq b$, then $a^2 > 0$
   (iii) $a + b = 0$, then $b = -a$
   (iv) $-(-a) = a$

   (b) Prove that there does not exist a rational number *r* such that $r^2 = 2$. 5

   (c) Show that the set of rational numbers is countable. Is every superset of a countable set countable? Justify. 6

2. (a) Find the supremum and infimum of the following sets if they exist: 4
   (i) $\left\{\frac{1}{n} : n \in \mathbb{N}\right\}$
   (ii) $\left\{(4n+3)/n : n \in \mathbb{N}\right\}$
   (iii) $(1,2] \cup [3,8)$
   (iv) $\left\{x \in 1 : x^2 \leq 25\right\}$

   (b) Prove that between any two distinct real numbers there always lies a rational number. 5

   (c) State and prove the Archimedean property of $\mathbb{R}$. 6

UNIT–II

3. (a) Define limit point of a set. Obtain the limit point and derived set of the set (a,b), where $a,b \in \mathbb{R}$. 5

   (b) Prove that every convergent sequence is bounded. Is the converse true? Justify. 5

   (c) Show that $\left\langle \sqrt{n^2+1} - \sqrt{n} \right\rangle$ is a null sequence. 5

4. (a) Define limit point of a sequence. Obtain the limit point of the sequence $\left\langle (-1)^n \right\rangle$. 4

   (b) Prove that the limit of the sum of two convergent sequences is the sum of their limits. 5

   (c) Show that the sequence $\langle f_n \rangle$ defined by $f_n = (1+1/n)^n$ is convergent and that $\lim_{n\to\infty}(1+1/n)^n$ lies between 2 and 3. 6

UNIT–III

5. (a) Prove that every subsequence of a convergent sequence is convergent. Is the converse true? Justify. 4

   (b) Show that every bounded sequence has a convergent subsequence. 5

   (c) If x_1, y_1 are two positive unequal numbers, $x_n = (x_{n-1} + y_{n-1})/2$ and $y_n = \sqrt{x_{n-1}y_{n-1}}$, $\forall n \geq 2$. Prove that the sequences $\langle x_n \rangle$ and $\langle y_n \rangle$ are monotonic and they converge to the same limit. 6

6. (a) If $\langle f_n \rangle$ be a sequence of positive numbers such that $f_n = (f_{n-1} + f_{n-2})/2, \forall n \geq 2$, then show that $\langle f_n \rangle$ converges. Also, find $\lim_{n\to\infty} f_n$. 5

   (b) Prove that a sequence converges if and only if it is a Cauchy sequence. 5

(c) Using Cauchy's criterion of convergence, examine the convergence of the sequence $\langle f_n \rangle$, where

$$f_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \ldots + \frac{1}{n!}.$$ 5

UNIT–IV

7. (a) Show that if the series $\sum_{n=1}^{\infty} u_n$ converges, then $\lim_{n\to\infty} u_n = 0$.

Show, by example that the converse is not true. 5

(b) Examine for convergence of the infinite series: 6

(i) $\sum_{n=1}^{\infty} \frac{1}{n^2 + a^2}$

(ii) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} + \sqrt{(n+1)}}$

(c) Use the D'Alembert's ratio test, to test the convergence of the series $\sum_{n=1}^{\infty} \frac{2^{n-1}}{3^n + 1}$. 4

8. (a) Test the convergence of the series: 6

(i) $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^{-n^2}$

(ii) $\sum_{n=1}^{\infty} 2^{-n-(-1)^n}$

(b) Show that $\sum_{n=1}^{\infty} \frac{1}{n^p}, p > 0$, is convergent if $p > 1$ and divergent if $p \le 1$. 5

(c) Test for absolute convergence of the series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n^2}{(n+1)!}$. 4
