October 2025
B.A./B.Sc.
Third Semester
MAJOR – 3
MATHEMATICS
Course Code: MAM 3.11
(Differential Equations)

Total Mark: 50 *Pass Mark: 20*
Time: 2 hours

I. Answer three *questions, taking* one *from each unit.*

UNIT–I

1. (a) Show that $x^3 + 3xy^2 = 1$ is an implicit solution of the differential equation $2xy\frac{dy}{dx} + x^2 + y^2 = 0$. 4

   (b) Solve the initial value problem
   $\left(2x\cos y + 3x^2\right)dx + \left(x^3 - 2x\sin y - y\right)dy = 0$, $y(0) = 0$ 4

   (c) Consider the differential equation $(4x + 3y)dx + 2xydy = 0$ then show that the equation is not exact and find the integrating factor. 4

2. (a) Solve the differential equation $y^2dx - (3xy - 1)dy = 0$ 4

   (b) Solve the initial value problem $x\sin y\, dx + (x+1)\cos y\, dy = 0$, $y(1) = \frac{\pi}{2}$ 4

   (c) Show the homogeneous equation
   $\left(Ax^2 + Bxy + Cy^2\right)dx + \left(Dx^2 + Exy + Fy^2\right)dy = 0$ is exact if and only if $B = 2D$ and $E = 2C$. 4

UNIT–II

3. (a) Formulate the general differential equation for exponential decay of radioactive element. 6

(b) Derive an expression for half-life time of a radioactive element. 2

(c) If you have 50 grams of ^{14}C today, how much will be left in 100 years? (Given ln 2 = 0.69314) 4

4. (a) Show the compartmental model and write down the balance law for the mass pollution in lake. 2

(b) Formulate the general differential equation of a lake pollution model. 4

(c) Formulate the pollution levels in the case study of Lake Burley Griffin. 6

UNIT–III

5. (a) Find the general solution of $\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 18y = 0$. 4

(b) Solve the initial-value problem $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 25y = 0$,

$y(0) = -3,\ y'(0) = -1$. 4

(c) Solve: $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2x^2 + e^x + 2xe^x + 4e^{3x}$ 4

6. (a) Given that e^{-x}, e^{3x} and e^{4x} are solutions of $x^3\frac{d^3y}{dx^3} - 4x^2\frac{d^2y}{dx^2} + 8x\frac{dy}{dx} - 8y = 0$. Show that they are linearly independent on the interval $-\infty < x < \infty$ and solve the general solution. 4

(b) Given that $y = x$ is a solution of $\left(x^2+1\right)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$.

Find a linearly independent solution by reducing the order. 4

(c) Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^x - \ln(\sin x)$, with conditions

$y(0) = 2,\ y'(0) = 4$. 4

II. Answer <u>any two</u> *of the following questions.*

7. (a) Solve: $\dfrac{dy}{dx} = xy^3$ 2

   (b) Solve: $(x-2y+1)dx+(4x-3y-6)dy=0$ 2

   (c) Solve the initial value problem
   $(3x-y-6)dx+(x+y+2)dy=0,\ y(2)=-2$ 3

8. (a) In a continuous population growth, find the equation for the time taken for the population to double in size. 2

   (b) Solve the differential equation $\dfrac{dX}{dt} = -\dfrac{r}{K}\left(X^2 - KX + \dfrac{Kh}{r}\right)$,
   when $r=1$, $K=10$, $h=\dfrac{9}{10}$, at $X(0)=x_0$. 2

   (c) Solve the differential equation $\dfrac{dC}{dt} = \dfrac{F}{V}(C_{in} - C)$, with the initial condition $C(0)=C_0$. 3

9. (a) Determine a word equation and appropriate compartment model for both prey and predator. 2

   (b) Formulate the differential equations for the predator-prey density model. 2

   (c) Find the equilibrium point of the differential equation.
   $\dfrac{dX}{dt} = \lambda_1 X - C_1 XY,\ \dfrac{dY}{dt} = C_2 XY - \lambda_2 Y$. 3
