October 2025
B.A./B.Sc.
First Semester
MAJOR – 1
MATHEMATICS
Course Code: MAM 1.11
(Calculus)

Total Mark: 50 *Pass Mark: 20*
Time: 2 hours

I. Answer <u>three</u> *questions, taking* <u>one</u> *from each unit.*

UNIT–I

1. (a) Evaluate any two of the following: 3×2=6

(i) $\lim_{x\to 0}(1+2x)^{-3/x}$

(ii) $\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{1}{x}\right)$

(iii) $\lim_{x\to 0^+}\frac{\ln x}{\csc x}$

(b) Sketch the graph of $r=\frac{1}{2}-\cos\theta$ in polar coordinates. 6

2. (a) Find the n^{th} derivative of any two of the following: 3×2=6

(i) $x^3\log x$

(ii) $(2x+3)^2\sin x$

(iii) $e^{2x-1}\cos x$

(b) A fertilizer producer finds that it can sell its product at a price of $p=300-0.1x$ dollars per unit when it produces x units of fertilizer. The total production cost (in dollars) for x units is $C(x)=15000+125x+0.025x^2$. If the production capacity of the firm is at most 1000 units of fertilizer in a specified time, how many units must be manufactured and sold in that time to maximize the profit? 6

UNIT–II

3. (a) Evaluate any two of the following: 3×2=6

(i) $\int \sin^3 x \cos^2 x \, dx$

(ii) $\int_0^{\pi/4} \sqrt{1+\cos 4x}\, dx$

(iii) $\int \tan^2 x \sec x \, dx$

(b) Find the volume of the cone generated when the triangle with vertices $(0,0)$, $(0,r)$, $(h,0)$ where $r>0$ and $h>0$, is revolved about the x-axis. 6

4. (a) Obtain the reduction formula for $\int \sin^n x \, dx$ and hence find $\int \sin^4 x \, dx$. 6

(b) Find the volume of the solid generated when the region enclosed $y=\sqrt{x+1}$, $y=\sqrt{2x}$ and $y=0$ is revolved about the x-axis. 6

UNIT–III

5. (a) Prove that $\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$, $-1<x<1$. Using this, obtain the derivative formula for $\tanh^{-1} x$. 6

(b) Prove any two of the following: 3×2=6

$\vec{r}(t), \vec{r}_1(t), \vec{r}_2(t)$ are vector valued functions in the parameter t and $f(t)$ is a real valued function in t.

(i) $\vec{r}(-t)$ and its derivative are orthogonal for all t.

(ii) $\frac{d}{dt}\left[f(x)\vec{r}(t)\right] = f(t)\frac{d}{dt}\left[\vec{r}(t)\right] + \frac{d}{dt}\left[f(t)\right]\vec{r}(t)$

(iii) $\frac{d}{dt}\left[\vec{r}_1(t)\cdot\vec{r}_2(t)\right] = \vec{r}_1(t)\cdot\frac{d}{dt}\vec{r}_2(t) + \frac{d}{dt}\vec{r}_1(t)\cdot\vec{r}_2(t)$

6. (a) Prove any two of the following: 3×2=6

(i) $\cosh 2x = \cosh^2 x + \sinh^2 x$

(ii) $\tanh 2x = \dfrac{2\tanh x}{1+\tanh^2 x}$

(iii) $\dfrac{d}{dx}\tanh x = \sec h^2 x$

(b) Suppose that the position vector of a particle at time t is $\vec{r}(t) = e^t\hat{i} + e^{-2t}\hat{j} + t\hat{k}$. 6

(i) Find the scalar tangential and normal component of acceleration at $t = 0$.

(ii) Find the vector tangential and normal component of acceleration at $t = 0$.

(iii) Find the curvature of the path at the point where the particle is located at $t = 0$.

II. Answer <u>any two</u> *of the following questions.*

7. Sketch the graph of any one of the following: 7

(a) $y = \dfrac{x^2 - 1}{x^3}$

(b) $y = (x-4)^{2/3}$

8. (a) Find the area of the surface generated by revolving the curve $y = x^3$, $0 \le x \le \dfrac{1}{2}$ about the x–axis. 4

(b) By revolving the semicircle $x = r\cos t$, $y = r\sin t$, $(0 \le t \le \pi)$ about the x-axis, show that the surface area of a sphere of radius r is $4\pi r^2$. 3

9. (a) A shell, fired from a cannon, has a muzzle speed of 800 ft/s. The barrel makes an angle of 45° with the horizontal and, for simplicity, the barrel opening is assumed to be at ground level. How far does the shell travel horizontally? 4

(b) Find the parametric equations of the line tangent to the graph of $\vec{r}(t) = \ln t\hat{i} + e^{-t}\hat{j} + t^3\hat{k}$ at the point where $t = 2$. 3
