October 2025
B.A./B.Sc.
Fifth Semester
DISCIPLINE SPECIFIC ELECTIVE – 2
MATHEMATICS
Course Code: MAD 5.21
(Boolean Algebra & Automata Theory)

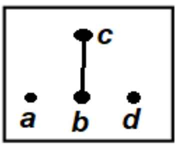
Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five *questions, taking* one *from each unit.*

UNIT–I

1. (a) Define POSET with example. 2
   (b) Draw the diagram of $M_3 \oplus M_4, 2^4, 1\oplus(1\cup 1)$. 3
   (c) Define ordered set. Show that $(\mathbb{N}, \geq)$ is a total ordered set. 1+3=4
   (d) Define order preserving and order embedding map between ordered sets with examples. Let $\varphi : P \to Q$ and $\psi : Q \to R$ be order preserving maps then show that the composite map $\psi o \varphi$ given $(\psi \circ \varphi)x = \psi(\varphi(x))$ for $x \in P$ is also order preserving map. 2+3=5

2. (a) Let P be a lattice, prove that for all $a, b, c, d \in P$. Then prove that: 5
   (i) $a \leq b \Rightarrow a \vee c \leq b \vee c$ and $a \wedge c \leq b \wedge c$
   (ii) $a \leq b$ and $c \leq d \Rightarrow a \vee c \leq b \vee d$ and $a \wedge c \leq b \wedge d$
   (b) Draw and label a diagram of the ordered sets $\mathcal{O}(P)$ of down sets for the ordered set P given by the diagram: 4



(c) Let $(L, \vee, \wedge)$ be a non-empty set equipped with two binary operations which satisfy the axioms of join and meet. Then: 5

(i) Prove that $\forall a, b \in L$ we've $a \vee b = b$ if $a \wedge b = a$.

(ii) Define $\leq$ on L by $a \leq b$ if $a \vee b = b$ then prove that $\leq$ is an ordered relation.

UNIT–II

3. (a) Let $f : B_1 \to B_2$ be a Boolean homomorphism. Then prove that $f(B_1)$ is a Boolean algebra and a Boolean sub-algebra of B_2. 5

(b) Prove that a lattice L is distributive if the cancelation rule holds. 5

(c) Show that (B, gcd, lcm) is a Boolean algebra if B is the set of all positive divisor of 110. 4

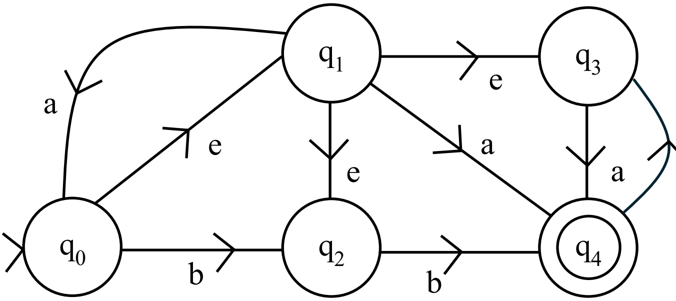
4. (a) Define disjunctive and conjunctive normal form with examples. 2

(b) A committee of three judges deciding on the acceptance or rejection of a competitor in a game are provided with buzzers in which the push is to indicate acceptance. Design a circuit so that a bell will ring when there is a majority of vote for acceptance. 6

(c) Minimize $xyz' + x'yz' + (x' + y'z')'(x + y + z')' + x(y + z)'$ using K-map and draw the contact diagram. 6

UNIT–III

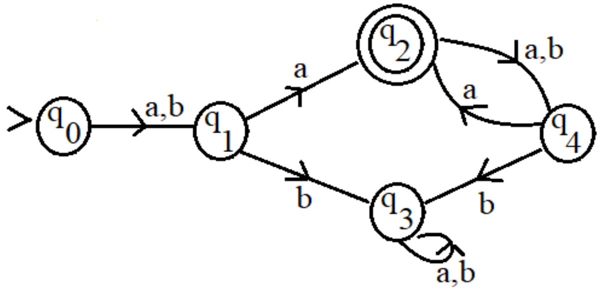
5. (a) Convert the given non-deterministic finite automata (NFA) to its equivalent deterministic finite automata (DFA). 4



(b) Show that the regular language is closed under union, concatenation, Kleen star, complementation, and intersection. 6

(c) Design a non-deterministic finite automaton (NFA) that accepts strings over $\{a,b\}^*$ which contains a substring *aa* or *bb*. 4

6. (a) Find the regular expression for the language accepted by the deterministic finite automaton whose state diagram is: 6



(b) Show that union and intersection of two regular language is regular. 4

(c) Show that $L=\{a^p : p \text{ is a prime}\}$ is not a regular language. 4

UNIT–IV

7. (a) Define a regular context free grammar. Construct a NFA for the CFG given by: 1+3=4

$V=\{S,A,B,a,b\};\sum=\{a,b\}$

$R=\{S\rightarrow bA; S\rightarrow aB; A\rightarrow abaS; B\rightarrow babS; S\rightarrow e\}$

(b) Show that CFG $G=(V,\Sigma,R,S)$ where $V=\{S,a,b,+,*\}$, $\sum=\{a,b,+,*\}$, $S=S$, and $R=\{S\rightarrow S+S; S\rightarrow S*S; S\rightarrow a; S\rightarrow b\}$ is an ambiguous grammar. 3

(c) Convert a grammar *G* to Greibach normal form, where the rules *R* of the Grammar *G* is given by: 4

$S\rightarrow AC/BB$

$B\rightarrow b/SB$

$C\rightarrow c$

$A\rightarrow b$

(d) Construct a PDA that accepts the language
$L = \{w \in \{a,b\}^* : w \text{ has the same number of } a\text{'s and } b\text{'s}\}$. 3

8. (a) Let G be a grammar with R given by:
$S \to aB / bA$
$A \to a / aS / bAA$
$B \to b / bS / aBB$
Then find the left most and rightmost derivation for the string *aaabbabbba*. Also, draw the parse tree. 4
(b) Prove the statement "Every non-deterministic finite automaton is equivalent to a push down automaton". 5
(c) Show that $L = \{ww : w \in \{0,1\}^*\}$ is not a context free language. 5

UNIT–V

9. (a) State the uses of Turing machine and give one example each with one sample computation. 4
(b) Construct a Turing machine which accept language: 4
$L = \{w \in \{a,b\}^* : a \text{ occur even number of times in } w \text{ and } |w| \text{ is even}\}$
(c) Construct a Turing machine that compute the function $f : \sum_0^* \to \sum_0^*$ define by $f(w) = ww^R$ and hence trace the input $\#abba\#$. 6

10. (a) Define a machine schema. Also, draw the copying machine and the right shifting standard machine. 1+4=5
(b) Find a post correspondence solution for the given list M = (110, 0011, 0110) and N = (110110, 00, 110). 3
(c) Define any two with example: 3×2=6
(i) Decidable
(ii) Undecidable
(iii) Post correspondence problem
(iv) Machine Schema