October 2025
B.A./B.Sc.
Fifth Semester
CORE – 11
MATHEMATICS
Course Code: MAC 5.11
(Multivariate Calculus)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five *questions, taking* one *from each unit.*

UNIT–I

1. (a) Define continuity of a function and show that

$$f(x,y)=\begin{cases}\dfrac{2xy}{x^2+y^2}, & (x,y)\neq(0,0),\\ 0, & (x,y)=(0,0),\end{cases}$$

is continuous at every point except at the origin. 5

(b) The linearization of $f(x,y)=x^2-xy+\frac{1}{2}y^2+3$ at (5, 4) is $L(x,y)=6x-y-10$. Find an upper bound for the error in the approximation $f(x,y)\approx L(x,y)$ over the rectangle $R:|x-5|\leq 0.1, |y-4|\leq 0.1$. 5

(c) Determine whether $u(x,y,z)=\left(x^2+y^2+z^2\right)^{\frac{1}{2}}$ is harmonic. 4

2. (a) Show that $\lim\limits_{(x,y)\to(0,0)}\dfrac{x^2y}{x^4+y^2}$ does not exist. 5

(b) The volume $V=\pi r^2h$, of a right circular cylindrical, is to be calculated from values of *r* and *h*. Suppose *r* is measured with an error of no more than 2% and h with an error of no more than 0.5%. Estimate the standard pointwise error of V. 5

(c) Calculate $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ using the following functions:

$z = f(x, y) = 3x^2 - 2xy + y^2, x = x(u, v) = 3u + 2v$

$y = y(u, v) = 4u - v$ 4

UNIT–II

3\. (a) For the function $f(x, y) = \frac{x^4}{2} + \frac{y^4}{2}$,

find the directions at the point $(2, 2)$ in which: 5

(i) f increases most rapidly

(ii) f decreases most rapidly

(iii) f shows zero change

(b) The surfaces $f(x, y, z) = x^2 + y^2 - z^2 = 0$,

$g(x, y, z) = x + z - 4 = 0$ intersect in an ellipse. Find the parametric equation of the tangent line at the point $P_0(3,1,1)$. 5

(c) Find the greatest and smallest values of $f(x, y) = xy$ subject to the ellipse $\frac{x^2}{8} + \frac{y^2}{2} = 1$. 4

4\. (a) Use the method of Lagrange multiplier to find: 7

(i) The minimum value of $x + y$ subject to the constraint $xy = 16, x > 0, y > 0$.

(ii) The maximum value of $x + y$, subject to the constraint $x + y = 16$.

(b) Find the extreme value of $f(x, y) = xy$ subject to the constraint $g(x, y) = x^2 + y^2 - 10 = 0$. 7

UNIT–III

5\. (a) Evaluate $\iint_R (x - y)\,dA$, where R is the region in the xy-plane with vertices (1, 0), (2, 0), (3, 2) and (0, 2). 5

(b) Find the surface area of the portion of the plane $x + y + z = 1$ that lies in the first octant. 5

(c) Find the volume of the tetrahedron T bounded by the plane $x + y + 3z = 6$ and the coordinate planes $x = 0, y = 0, z = 0$. 4

6. (a) Find the volume of the solid D bounded below by the paraboloid $z = x^2 + y^2$ and above by the plane $x + z = 3$. 5

(b) Find the volume of the solid in the first octant that is bounded by the cylinder $x^2 + y^2 = 2y$, the half cone $z = \sqrt{x^2 + y^2}$ and the xy-planes. 5

(c) Evaluate $I = \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{-\sqrt{2-x^2-y^2}}^{\sqrt{2-x^2-y^2}} x\, dz\, dy\, dx$ by changing to cylindrical or spherical coordinates. 4

UNIT–IV

7. (a) If $u = xy$ and $v = x^2 - y^2$, find the Jacobian $\dfrac{\partial(x,y)}{\partial(u,v)}$ and express it in terms of u and v. 4

(b) Find the area of the region E bounded by the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$. 5

(c) Evaluate the line integral $\int_c xy\, ds$, where C consists of: 5

(i) The line segment C_1 from $(-3, 3)$ to $(0, 0)$

(ii) The curve $C_2 : 16y = x^4$ from $(0, 0)$ to $(2, 1)$

8. (a) An object moves in the force field $\vec{F} = y^2\hat{i} + 2(x+1)y\hat{j}$. How much work is performed as the object moves from the point $(2, 0)$ counterclockwise along the elliptical path $x^2 + 4y^2 = 4$ to $(0, 1)$, then back to $(2, 0)$ along the line segment joining the two points. 5

(b) Evaluate the line integral $\int_C \vec{F} \cdot dr$, where $\vec{F} = \vec{\nabla}\left(e^x \sin y - xy - 2y\right)$ and C is the path described by $\vec{R}(t) = \left[t^3 \sin\left(\frac{\pi}{2}t\right)\right]\hat{i} + \left[\frac{1}{2}\cos\left(\pi t + t\right)\right]\hat{j}$ for $0 \le t \le 1$. 5

(c) Show that the vector field $\vec{F} = (20x^3z + 2y^2, 4xy, 5x^4 + 3z^2)$ is conservative in $\mathbb{R}^3$ and find a scalar potential function for $\vec{F}$. 4

UNIT–V

9. (a) State and prove Green's theorem. 5

(b) Find the flux of the vector field $\vec{F} = z\hat{i} + x\hat{j} + \left(y+z\right)\hat{k}$ through the parameterized surface $\vec{R}(u,v) = (uv)\hat{i} + (u-v)\hat{j} + (2u+v)\hat{k}$ over the triangular region D in the uv-plane that is bounded by $u = 0$, $v = 0$, and $u + v = 1$. 5

(c) Show that Green's theorem is true for the line integral $\oint_C (-y\,dx + x\,dy)$, where C is the closed path. 4

10. (a) Evaluate $\oint_C \left(y^2 dx + z\,dy + x\,dz\right)$, where C is the curve of intersection of the plane $x + z = 1$ and the ellipsoid $x^2 + 2y^2 + z^2 = 1$, oriented counterclockwise. 7

(b) Let $\vec{F} = 2x\hat{i} - 3y\hat{j} + 5z\hat{k}$, and let S be the hemisphere $z = \sqrt{9 - x^2 - y^2}$ together with the disk $x^2 + y^2 \le 9$ in the xy-plane. Verify the divergence theorem. 7