2023
M.Sc.
First Semester
CORE – 03
PHYSICS
Course Code: MPHC 1.31
(Mathematical Physics)

Total Mark: 70   *Pass Mark: 28*
Time: 3 hours

Answer <u>five</u> questions, taking <u>one</u> from each unit.

UNIT–I

1. (a) Prove the Fourier integral theorem.   6
   (b) Find the Fourier transform of:

   $$f(x) = \begin{cases} 1 - x^2 & ,if \ |x| \leq 1 \\ 0 & ,if \ |x| > 1 \end{cases}$$

   and use it to evaluate $\int_0^\infty \left(\dfrac{x\cos x - \sin x}{x^3} \right) \cos \dfrac{x}{2} \, dx$.   8

2. (a) Find the Fourier transform of the function:

   $$f(x) = \begin{cases} \dfrac{1}{2\in} & ,if \ |x| \leq \in \\ 0 & ,if \ x > \in \end{cases}$$
      2

   (b) Using Parseval's identity, show that $\int_0^\infty \dfrac{x^2 dx}{(x^2 + 1)} = \dfrac{\pi}{4}$.   4

   (c) Use Fourier sine transform to solve the equation: $\dfrac{\partial u}{\partial t} = k \dfrac{\partial^2 u}{\partial x^2}$ under

   the conditions $u(0, t) = 0$, $u(x, 0) = e^{-x}$, $u(x, t)$ is bounded.   8

UNIT–II

3. (a) Find the Laplace transform of $\dfrac{\sin at}{t}$.   2

    (b) Obtain the inverse Laplace transform of $\dfrac{1}{(s+2)^2(s-2)}$.   4

    (c) Using the Laplace transform, find the current $i(t)$ in LC circuit. Assume $L = 1$ H and $C = 1$ F under the condition:
$$V(t) = \begin{cases} t, & \text{if } 0 < t < 1 \\ 0, & \text{otherwise} \end{cases}$$
   8

4. (a) Find the inverse Laplace transform of $\dfrac{s+1}{s^2-6s+25}$.   2

    (b) Evaluate $\displaystyle\int_0^\infty \dfrac{e^{-at} - e^{-bt}}{t}\, dt$.   4

    (c) An inductor of 3 henry is in series with a resistance of 30 ohms and an emf of 150 volts. Assume that the current is zero at $t = 0$, find the current for time $t > 0$.   8

UNIT–III

5. (a) Show that the metric tensor g_{ij} is a covariant symmetric tensor of rank two.   4

    (b) Write a short note on the following: 2×2=4
      (i) Covariant tensor
      (ii) Contravariant tensor

    (c) Prove that an anti-symmetric tensor A_{ij} has $\dfrac{n}{2}(n-1)$ independent components.   6

6. (a) Explain inner product of tensors with an example involving the process.   6

    (b) Prove that the Christoffel symbols are not tensor quantity.   8

UNIT–IV

7. (a) Express $f(x) = 1 + x - x^2$ in terms of Legendre polynomials.   4

   (b) Prove that $J_{5/2}(x) = \sqrt{\dfrac{2}{\pi x}}\left[\left(\dfrac{3-x^2}{x^2}\right)\sin x - \dfrac{3\cos x}{x}\right].$ 4

   (c) Using the descending power series of $y'' - 2xy' + 2ny = 0$, arrive at the Hermite's polynomial.   6

8. (a) Write the trigonometric expansion involving Bessel functions from its generating function.   4

   (b) Prove that $P'_{n+1} - P'_{n-1} = (2n+1)P_n$.   4

   (c) Derive Neumann function using the equation
   $$x^2 y'' + xy' + (x^2 - n^2)y = 0.$$ 6

UNIT–V

9. (a) Find the centre of $G = \{e, a, b, ab\}$.   4

   (b) Let $<G,*>$ and $<G',(\bmod 3)>$ be two groups where

   $G = \{1, \omega, \omega^2\}$ and $G' = \{0, 1, 2\}$. Show that $G \cong G'$.   5

   (c) Write the symmetry operations involving point groups.   5

10. (a) Explain proper and improper rotation with examples.   4

    (b) Find all the generators of the cyclic group $\{a, a^2, a^3, a^4 = e\}$.   4

    (c) Discuss the consequences of great orthogonality theorem.   6

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