2023
M.Sc.
Third Semester
DISCIPLINE SPECIFIC ELECTIVE – 01
MATHEMATICS
Course Code: MMAD 3.11
(Classical Mechanics)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) Define the degrees of freedom of a dynamical system. Determine the degrees of freedom in the following cases: 1×5=5
   (i) Five particles are moving freely in a plane.
   (ii) A rigid body is moving in space with one point fixed.
   (iii) A particle is moving on the circumference of a circle.
   (iv) A rigid body is moving freely in a three dimensional space.
   (b) Explain conservative and non-conservative forces with examples. 4
   (c) A bead of slides on a smooth rod which is rotating about one end in a vertical plane with uniform angular velocity ω. Show that the equation of motion is $m\ddot{r} = mr\omega^2 - mg\sin\omega t$. 5

2. (a) State Hamilton's principle. Derive Lagrange's equations of motion from Hamilton's principle. 7
   (b) If L is the Lagrangian for a system of n degrees of freedom, satisfying Lagrange equations, show by direct substitution that $L' = L + \frac{dF}{dt}$ also satisfies Lagrange equations, where $F = F(q_1, q_2, \ldots, q_n, t)$ is any arbitrary and differentiable function of its argument. 7

UNIT–II

3. (a) Define the following: 1×3=3
   (i) Generalized momenta (ii) Cyclic coordinates
   (iii) Routhian
   (b) Show that if generalised coordinate q_k is a cyclic coordinate then generalised momenta p_k is constant in any motion. 3
   (c) The Lagrangian for a system can be written as

   $$L = a\dot{x}^2 + b\frac{\dot{y}}{x} + c\dot{x}\dot{y} + fy^2\dot{x}\dot{y} + g\dot{y}^2 - k\sqrt{x^2+y^2}$$, where a, b, c, f, g and k are constants. What is the Hamiltonian? Which quantities are conserved? 6+1+1=8

4. (a) Derive Hamilton's equations in spherical coordinate system. 7
   (b) Find the differential equation of a moving particle under the action of a central force field. 7

UNIT–III

5. (a) Derive Hamilton's principle for non-conservative holonomic system from D'Alembert's principle and hence deduce Hamilton's principle for conservative holonomic system. 5+2=7
   (b) Find the equation of motion and force of constraints in case of a simple pendulum by using Lagrange's method of undetermined multipliers. 7

6. (a) State the principle of least action. Derive Jacobi's form of principle of least action. 1+6=7
   (b) By using Lagrange's method of undetermined multipliers find the equation of motion of a hoop rolling down an inclined plane without slipping. 7

UNIT–IV

7. (a) What are canonical transformations? Show that the transformations

   $Q = \ln\left(\frac{\sin p}{q}\right)$ and $P = q\cot p$ are canonical and also obtain the generating function $F_1(q, Q)$ 2+2+3=7

(b) Write down the condition for a transformation to be canonical. Discuss how the transformation equations can be obtained from generating functions of type F_1 and F_2. 1+3+3=7

8. (a) Obtain the bilinear invariant condition for a transformation to be canonical. 7

(b) Show that the Lagrange's bracket is invariant under canonical transformation. 7

UNIT–V

9. (a) Find the relation between the angular momentum vector and the inertia tensor. 3½+3½=7

(b) Obtain Euler's equations of motion for a rigid body. 7

10. (a) Discuss the motion of a heavy symmetrical top. 8

(b) Write short notes on the following: 2×3=6

(i) Nutation

(ii) Gyroscope

(iii) Principal axes theorem