2023
M.Sc.
Third Semester
CORE – 10
MATHEMATICS
Course Code: MMAC 3.21
(Topology)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) Show that if (X,τ) be a topological space and $Y \subseteq X$, then the collection $\tau_Y = \{Y \cap M : M \in \tau\}$ is a subspace topology on Y. 5

   (b) If (X,τ) and be a topological space; $A, B \subseteq$ X then show that

   (i) $(A \cap B)' \subseteq A' \cap B'$

   (ii) $\overline{(A \cup B)} \subseteq \overline{A} \cup \overline{B}$ 2½+2½=5

   (c) Let X be a nonempty set. Let τ and τ' be two topologies on X, then

   (i) Is $\tau \cup \tau'$ a topology?

   (ii) Is $\tau \cap \tau'$ a topology? 2+2=4

2. (a) Let (X,τ) be a topological space and $A \subseteq X$ then show that A is closed if and only if $A' \subseteq A$ 5

   (b) Let $\mathcal{B}$ and $\mathcal{B}'$ be bases for the topologies τ and τ' respectively, on X.
   Then prove that the following statements are equivalent

   (i) τ' is finer than τ

   (ii) For each $x \in X$ and each basis $B \in \mathcal{B}$ containing x, there is a basis element $B' \in \mathcal{B}'$ such that $x \in B' \subseteq B$. 5

   (c) Let (X,τ) be a topological space and $A \subseteq X$ then show that

   $\overline{A} = A^{\circ} \cup bd(A)$ 4

UNIT–II

3. (a) Let (X,τ) and (Y,τ') be two topological spaces. Prove that the function $f: X \to Y$ is continuous if and only each open set V in τ', $f^{-1}(V)$ an open set in τ. 5

   (b) State the pasting lemma. Give an elaboration of the lemma with an example. 1+4=5

   (c) Show that $P = [a, b]$ is homeomorphic to $Q = [c, d]$; P, Q are subspace of $\mathbb{R}$ with the standard topology. 4

4. (a) Let (X,τ) and (Y,τ') be two topological spaces. Prove that $f: X \to Y$ is continuous if and only if $f(\overline{A}) \subseteq \overline{f(A)}$ for every subset A and X. 5

   (b) Homeomorphism forms an equivalence class on the class of all topological spaces. Justify. 5

   (c) Let (X,τ) and (Y,τ') be two topological spaces. Prove that $f: X \to Y$ is open if and only if $f(A^\circ) \subseteq (f(A))^\circ$. 4

UNIT–III

5. (a) If E is a connected subset of a disconnected topological space (X,τ), the sets A and B form a separation of E, then show that either $E \subseteq A$ or $E \subseteq B$. 5

   (b) A connected topological space may not be path connected. Justify. 5

   (c) If $A \subseteq \mathbb{R}$ is connected, then A is an interval. Justify. 4

6. (a) If E is a connected subset of a topological space (X,τ), then show that $F \subseteq X$ such that $E \subseteq F \subseteq \overline{E}$ is connected. 5

   (b) Prove that a topological space (X,τ) is locally connected if and only if components of each open subset of X are open. 5

   (c) Is a product of path connected spaces necessarily path connected? 4

UNIT-IV

7. (a) Prove that the continuous image of a compact topological space is compact. 5

   (b) Prove that [0, 1] is a compact subset of $\mathbb{R}$ with the standard topology. 5

   (c) A subspace of a locally compact topological space is not locally compact. Justify. 4

8. (a) Let (X, τ) be a topological space. Then, prove that X is compact if and only if for every collection C of closed sets in X satisfy the finite intersection property. 6

   (b) Let (Y, τ') be a one-point compactification of a locally compact Hausdorff topological space (X, τ). Then, prove that (Y, τ') is Hausdorff. 4

   (c) A compact set in a topological space is not necessarily closed. Justify. 4

UNIT-V

9. (a) Prove that every compact Hausdorff space is normal. 5

   (b) A subspace of a first countable space is first countable. Justify. 5

   (c) The space $(\mathbb{R}, \tau_{std})$ is a T_3-space. Justify. 4

10. (a) Prove that every second countable space is first countable. Is the converse of the statement true? 5

    (b) A metrizable space is a normal space. Justify. 5

    (c) The closed image of a T_1-space is a T_1- space. Justify. 4
