2023
M.Sc.
Third Semester
CORE – 09
MATHEMATICS
Course Code: MMAC 3.11
(Numerical Analysis)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) Write a short notes on errors on numerical methods. 3
   (b) Discuss the rate of convergence of Regula-Falsi method. 5
   (c) Perform four iteration of the Newton Raphson method to obtain the root of the equation $f(x) \equiv \sin x = 1 + x^3$ which lies in the interval $(-2, -1)$. 6

2. (a) What is an iterative method? What are the criterion for termination of iterative methods? 2
   (b) Find the root of the equation $f(x) \equiv x^3 - x^2 - x + 1 = 0$ by performing three iteration of the Muller's method and taking $x_0 = 0.6, x_1 = 0.8$ and $x_2 = 1.2$. 6
   (c) Perform three iteration of the multipoint method to find the smallest positive root of the equation $f(x) \equiv x^3 - 5x + 1 = 0$. 6

UNIT–II

3. (a) Solve the system of equations by matrix inversion method 7

$$4x - y = 1$$
$$-x + 4y - z = 0$$
$$-y + 4z = 0$$

(b) Solve the system of equation by LU decomposition method taking

$u_{ii} = 2$ for $i = 1, 2, 3$ 7

$$x + y - z = 2$$

$$2x + 3y + 5z = -3$$

$$3x + 2y - 3z = 6$$

4. (a) By using Gauss-Seidel method solve the system of equations 6

$$\begin{bmatrix} 4 & 1 & 2 \\ 3 & 5 & 1 \\ 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \\ 3 \end{bmatrix}$$

(b) Find the eigenvalue which is nearest to 5 for the matrix 8

$$\begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 4 \end{bmatrix}$$

UNIT–III

5. (a) Deduce the Lagrange interpolating polynomial for the data 5

x	0	3	4	5	7	10
$f(x)$	3	31	69	131	351	1011

(b) Construct the interpolating polynomial that fits the data 5

x	0.0	0.1	0.2	0.3	0.4	0.5
$f(x)$	–1.5	–1.27	–0.98	–0.63	–0.22	0.25

using Newton's backward interpolation and hence estimate the values of $f(x)$ at 0.15, 0.45

(c) Prove the following: 1×3=3

(i) $\mu^2 = 1 + \frac{1}{4}\delta^2$

(ii) $\Delta(f(x)/g(x)) = (g(x)\Delta f(x) - f(x)\Delta g(x)/(g(x)g(x+h))$

(iii) $(1+\Delta)(1-\nabla) = 1$

(d) Compute: $\Delta^3(1-2x)(1-3x)(1-4x)$ 1

6. (a) Construct the Hermite interpolation polynomial that fits the data 7

x	$f(x)$	$f'(x)$
1	7.389	14.778
2	54.598	109.196

Estimate the value of $f(x)$ at $x = 1.5$, if the data is representing the function $f(x) = e^{2x}$, find the absolute error at $x = 1.5$

(b) For the following data, obtain the piecewise quadratic interpolating polynomials 7

x	–2	0	1	3	4
$f(x)$	–23	1	4	82	193

Interpolate at $x = -0.5$ and $x = 2.0$

UNIT–IV

7. (a) Find the first two derivatives of $f(x)$ at $x = 1$ from the following 6

x	–2	–1	0	1	2	3	4
$f(x)$	104	17	0	–1	8	69	272

(b) Using divided difference, find the value of $f''(8)$, given that $f(6) = 1.556$, $f(7) = 1.690$, $f(9) = 1.908$, $f(12) = 2.158$ 8

8. (a) Find the value of $I = \int_1^2 \frac{dx}{3x+5}$ using Simpson's 1/3 rule with 8 subinterval. Compare with the exact solution and find the absolute errors in the solutions. 7

(b) Find the value of the integral $I = \int_2^3 \frac{\cos 2x}{1+\sin x} dx$ using the Gauss Legendre two and three points integration rule. 7

UNIT–V

9. (a) Reduce the system of higher order initial value problem into a system of first order differential equations

$$u''' + e^x u'' + v'' + u + v = \sin x, u(0) = 0, u'(0) = 1, u''(0) = 2$$

$$v''' + \cos xv'' + \log x(u' + v') + u + 3v = 9, v(0) = 3, v'(0) = 2, v(0) = 0$$

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(b) Use Picard's method to solve $\frac{dy}{dx} = 1 + xy$, with $x_0 = 2, y_0 = 0$ 5

(c) Solve the initial value problem $\frac{dy}{dx} = 2x + 3y, y(0) = 1$, using modified Euler's method with $h = 0.25$ over the interval [0,1]. Compare with the exact solution. **6**

10. (a) Find the solution at $t = 0.5$ for the initial value problem $y' = t - y^2, y(0) = 1$ by the Adam-Bashforth method of order three with h = 0.1. Determine the starting values using a second order Runge-Kutta method. 7

(b) Find the solution at $t = 0.2$ for the initial value problem $u' = t^2 - 3u, u(0) = 2$ using Adam-Moulton third order method with $h = 0.1$. 7