2023
B.A./B.Sc.
Third Semester
GENERIC ELECTIVE – 3
STATISTICS
Course Code: STG 3.11
(Basics of Statistical Inference)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) Define *any three* of the following: 1×3=3
   (i) Parameter (ii) Statistic (iii) Estimator
   (iv) Estimate (v) P-value

   (b) Prove that if T_n is a consistent estimator of $\gamma(\theta)$ and $\psi\{\gamma(\theta)\}$ is a continuous function of $\gamma(\theta)$, then $\psi(T_n)$ is a consistent estimator of $\psi\{\gamma(\theta)\}$. 5

   (c) A random sample $(X_1, X_2, X_3, X_4, X_5)$ of size 5 is drawn from a normal population with unknown mean μ. Consider the following estimators to estimate μ:

   (i) $t_1 = \dfrac{X_1 + X_2 + X_3 + X_4 + X_5}{5}$

   (ii) $t_2 = \dfrac{X_1 + X_2}{2} + X_3$

   (iii) $t_3 = \dfrac{2X_1 + X_2 + \lambda X_3}{3}$

   Where λ is such that is an unbiased estimator of μ? Find λ. Are t_1 and t_2 unbiased? State giving reasons, the estimator which is best among t_1, t_2, and t_3. 6

2. (a) What is a minimum variance unbiased estimator? 2

(b) Define a sufficiency estimator . Let $x_1, x_2, \ldots, x_n$ be a random sample from $N(\mu, \sigma^2)$ population. Find sufficient estimators for μ and σ^2. 2+4=6

(c) If T_1 and T_2 are two unbiased estimator of $\gamma(\theta)$, having the same variance and ρ is the correlation between them, then show that $\rho \geq 2e-1$, where e is the efficiency of each estimator. 6

UNIT–II

3. (a) Out of the two types of error in testing the more severe error is 1

(i) type-I error (ii) type-II error
(iii) no error is severe (iv) both (i) and (ii)

(b) Define the null and alternative hypothesis with one example each. 4

(c) Explain the two types of errors and show the four possible situations that arise in any test procedure relating to decision and hypothesis. 4

(d) Given the frequency function:

$$f(x,\theta) = \begin{cases} \frac{1}{\theta} & , 0 \leq x \leq \theta \\ 0 & , elsewhere \end{cases}$$

and that you are testing the null hypothesis $H_0 : \theta = 1$ against $H_1 : \theta = 2$, by means of a single observed value of *x*. What would be the sizes of the type-I and type-II errors, if you choose the interval (i) $0.5 \leq x$, (ii) $1 \leq x \leq 1.5$ as the critical regions? Also obtain the power function of the test. 5

4. (a) Size of the critical region is known as 1

(i) power of the test (ii) size of type-II error
(iii) critical value of the test statistics (iv) size of the test

(b) Discuss Neyman-Pearson lemma for the best critical region. 7

(c) If $x \geq 1$ is the critical region for testing $H_0 : \theta = 2$ against the alternative $\theta = 1$, on the basis of the single observation from the population, $f(x,\theta) = \theta \exp(-\theta x), 0 \leq x \leq \infty$. Obtain the values of type-I and the power function of the test . 6

UNIT–III

5. (a) Student's *t*-test was invented by 1
   (i) R.A. Fisher (ii) W.S. Gosset
   (ii) W.G. Cochran (iv) G.W. Snedecor

   (b) Describe how χ^2-test may be used to test the independence of attributes. 7

   (c) Explain *F*-test for testing the equality of two population variances. 6

6. (a) Degrees of freedom for statistic-χ^2 in case of contingency table of order (2×2) is 1
   (i) 3 (ii) 4 (iii) 2 (iv) 1

   (b) A sample of 900 members has a mean 3.4 cms and standard deviation 2.61 cms is the sample from a large population of mean 3.25 cms and standard deviation 2.61 cms? If the population is normal and is unknown, find the 95% and 98% fiducial limits of true mean. (Table value 95% = 1.96, 98% = 2.33) 6

   (c) Define student's *t*-distribution. Write down the applications of *t*-distribution stating its assumptions. 2+5=7

UNIT–IV

7. (a) Which one of the following is not true for probability sampling? 1
   (i) Each unit has an equal chance of being selected.
   (ii) The sample is selected with a definite purpose in view.
   (iii) Sampling units have different probabilities of being selected.
   (iv) A sampling unit may be assigned the probability of selection in proportional to its size.

   (b) Describe the basic principles of sample survey. 6

   (c) Define SRSWOR. Show that in simple random sampling without replacement, the sample mean square is an unbiased estimate of the population mean square i.e. $E\left(s^2\right) = S^2$. 2+5=7

8. (a) The sample of all possible samples of size 2 from a population of 4 units is 1
   (i) 2 (ii) 4 (iii) 6 (iv) 8

   (b) Write a notes on sampling error and non-sampling error. 6

(c) Define simple random sampling. Show that in simple random sampling without replacement $V(\overline{x}_n)$ is given by

$$V(\overline{x}_n) = \frac{N-n}{N} \cdot \frac{S^2}{n}.$$ 2+5=7

UNIT–V

9. (a) Define analysis of variance (ANOVA). Also give the classified data for one way ANOVA along with ANOVA table for testing of hypothesis. 1+2+2=5

(b) In design of experiment define the following terms: 2×2=4

(i) Treatment (ii) Plot

(c) Define randomized block design (RBD). Also give the statistical analysis of RBD. 1+4=5

10. (a) Explain in details various principles involved in design of experiment. 6

(b) Give the statistical analysis of two way ANOVA and obtain the parameters by applying principles of least squares. 5

(c) Give the lay out of completely randomized block design (CRD). 3