2023
B.A./B.Sc.
Fifth Semester
CORE – 11
STATISTICS
Course Code: STC 5.11
(Stochastic Processes & Queuing Theory)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) Define probability generating function (p.g.f.) and express mean and variance in terms of p.g.f. 2+4=6

(b) Let Y have the distribution of geometric form given by,

$$P_r(Y=k)=g^{k-1}.p;k=1,2,3,\ldots$$

Show that the p.g.f. of Y is $\frac{ps}{1-qs}$ and $E(Y)=\frac{1}{p}$ and

$$Var(Y)=\frac{q}{p^2}$$ 5

(c) If $P(s)$ is the p.g.f. of X, find the p.g.f. of $\frac{x-m}{n}$, where m and n are any arbitrary constants. 3

2. (a) Define the following: 2+3=5
   (i) Bivariate probability generating function
   (ii) Stochastic process with example

(b) Suppose X be a Poisson variate with probability generating function $P(s)=e^{\lambda(s-1)}$, then find $p_k^{/}s$; where $k=1,2,3,\ldots$ 6

(c) Write a note on specification of stochastic process. 3

UNIT–II

3. (a) Define the following: 2×2=4
   (i) Markov chain
   (ii) Transition matrix
   (b) State and prove first entrance theorem. 2+4=6
   (c) For the given Markov chain having transition probability matrix $\begin{pmatrix} 1-a & a \\ b & 1-b \end{pmatrix}$, derive for the generalization of independent Bernoulli trials. 4

4. (a) For a higher transition probability, state and prove Chapman-Kolmogorov equation. 7
   (b) Define the following: 1½×2=3
   (i) Stability of Markov chain
   (ii) Transient and Persistent state
   (c) Define graph theoretic approach and write the formula for $\{V_k\}$. 4

UNIT–III

5. (a) Write down the three postulates of Poisson process. With usual notation show that under these postulates, number of occurrences $N(t)$ follows Poisson probability law. 3+7=10
   (b) What is pure death process? Write down the three axioms of pure death process. 1+3=4

6. (a) Show that if $M(t)$ is the number of occurrences recorded in an interval of length t, from a Poisson process $\{N(t), t \geq 0\}$, then $M(t)$ is also a Poisson process with parameter λpt, where λt is the parameter of $N(t)$. 6
   (b) State and prove Yule-Furry process. 8

UNIT–IV

7. (a) Define a queueing system. What are the different behaviours of customers in a queueing system? Define traffic intensity. 2+2+2=6
   (b) State and prove the theorem for distribution of arrival in a queueing system. 8

8. (a) Show that in a queueing system the number of departures in time t follows the truncated Poisson distribution. 8

(b) Obtain the two steady state difference equations for model 1: (M|M|1):(∞ |FCFS), i.e., birth and death model. 6

UNIT–V

9. (a) Explain the income and price elasticities of demand of a commodity. Discuss the general principles regarding the elasticity of demand. 6+4=10

(b) Let the demand curve for a commodity be $d = a - bp$ and the supply curve be $s = cp - d$, where a, b, c and d are positive constants. Find the equilibrium price and the quality exchanged. 4

10. (a) State Pareto's law of income distribution. 2

(b) Write notes on the following: 4×3=12

(i) Utility function

(ii) Curves of concentration in demand analysis

(iii) Engel's law