2023
B.A./B.Sc.
Third Semester
CORE – 7
STATISTICS
Course Code: STC 3.31
(Mathematical Analysis)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five *questions, taking* one *from each unit.*

UNIT–I

1. (a) Represent $\sqrt{9.3}$ on a number line. 3
   (b) Find the supremum and infimum of the set, if they exist. 2½×2=5
   (i) $S_n = \left\{ (-1)^n . \frac{1}{n} : n \in N \right\}$
   (ii) $S_n = \left\{ \frac{4n+3}{n} : n \in N \right\}$
   (c) Define limit point of a sequence. Prove that every point of [0, 1] is a limit point of (0, 1). 2+4=6

2. (a) Prove that a point P is a limit point of a set S if and only if every neighbourhood of P contains infinitely many points of S. 5
   (b) Show that the set $[3,4] \cup [5,6]$ is a closed set. 2
   (c) Show whether the given sequence is convergent, divergent or oscillatory.
   $\langle a_n \rangle = \left\langle (-1)^n + \frac{1}{2^n} \right\rangle$ 2
   (d) Define neighbourhood of a point. Prove that any open interval $]a, b[$ is a neighbourhood of each of its point. 2+3=5

UNIT–II

3. (a) State and prove D'Alembert ratio test. 6

   (b) Test the convergence of the series whose n^{th} term is

   $\left[\sqrt{(n^4+1)}-\sqrt{(n^4-1)}\right]$. 3

   (c) Test the convergence of the series $\sum\left[\frac{\log n}{\log(n+1)}\right]^{n^2 \log n}$ 3

   (d) Show that the series $1+\frac{1}{2!}+\frac{1}{3!}+\ldots$ is convergent. 2

4. (a) State comparison test. 2

   (b) Apply Cauchy's condensation test to discuss the convergence of the series $\sum_{n=2}^{\infty}\frac{1}{(n\log n)(\log\log n)^p}$. 3

   (c) Test the convergence of the series

   $\frac{1}{x}-\frac{1}{x+a}+\frac{1}{x+2a}+\ldots, x>0, a>0$. 3

   (d) Define the following: 2×2=4

   (i) Absolute convergence

   (ii) Conditional convergence

   (e) Use Cauchy's integral test to test the convergence of

   $1+\frac{1}{2^2}+\frac{1}{3^2}+\ldots$. 2

UNIT–III

5. (a) Show whether the given function exists at

   $f(x)=\begin{cases}5x-4, & 0<x\le 1\\ 4x^3-3x, & 1<x<2\end{cases}$ at x=1. 4

   (b) Define Rolle's mean value theorem and show its geometrical interpretation. Verify the Rolle's theorem for the function for

   $f(x)=x^2-3x+2$ on $[1, 2]$. 3+4=7

(c) Find the expansion of $\sin x$. 3

6. (a) Verify Lagrange's value theorem for $f(x) = x + \frac{1}{x}$ on $[1, 3]$ 4

(b) Show whether the function $f(x) = \frac{1}{x}, x \in (b, \infty), b > 0, x, y \in \mathbb{R}$ and $|x - y| < \delta$, is uniformly continuous or not. 3

(c) Expand $e^x \cos x$ in power of x by Maclaurin's theorem. 3

(d) Examine the function for continuity at the origin

$$f(x) = \begin{cases} \frac{4x^2 - 1}{2x + 1}, & if\ x \neq \frac{-1}{2} \\ -2, & f\ x = \frac{-1}{2} \end{cases}. \quad 4$$

UNIT–IV

7. (a) Define operator Δ. Evaluate $\Delta^2(x^2 + 1)$. 1+3=4

(b) State and prove Newton's divided difference interpolation formula. 5

(c) Given

x :	0	1	4	6
$f(x)$:	2	6	30	56

Obtain the value of $f(3)$ by Lagrange's interpolation formula. 5

8. (a) Estimate the missing entries in the table

x :	1	2	3	4	5
$f(x)$:	7	___	13	___	37

4

(b) State and prove Gauss's backward central difference formula. 5

(c) State Newton forward interpolation formula. Using this formula find $f(x)$ from the following data $f(0) = 8, f(2) = 11, f(4) = 20, f(6) = 41$. 5

UNIT–V

9. (a) State and prove Weddle's rule of numerical integration. 5

(b) Obtain the approximate quadrature formula

$$\int_{1}^{-1} U_x dx = \frac{1}{12}[13(U_1 + U_{-1}) - (U_3 + U_{-3})] \quad 5$$

(c) Fill in the blank: 1

(i) The iterative method $x = g(x)$ is _____ if $|g'(x)| < 1$.

(d) Show that the following rearrangement equation

$x^3 + 6x^2 + 10x - 20 = 0$, does not yield a convergent sequence of successive approximation by iteration method near $x = 1$. 3

10. (a) In trapezoidal rule, what is the degree of polynomial for $f(x)$? Derive Simpson's one-third rule formula for numerical integration. 1+5=6

(b) Find the reciprocal of 41 correct to 4 decimal places by iterative formula $x_{i+1} = x_i(2 - 41x_i)$. 4

(c) Explain Regula-Falsi method of finding the real roots of the equation $f(x) = 0.4$. 4
