2023
B.A./B.Sc.
Third Semester
CORE – 5
STATISTICS
Course Code: STC 3.11
(Sampling Distributions)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) Define the following: 2×2=4
   (i) Convergence in probability (ii) Convergence in mean square
   (b) State and prove De-Moivre Laplace theorem. 7
   (c) Let X_n be a sequence of random variables such that

$$X_n(x) = \begin{cases} \left(1-\frac{1}{n}\right)^{nx}; & x > 0 \\ 0; & \text{otherwise} \end{cases}$$

   Show that X_n converges is distribution to exponential. 3

2. (a) Define the following: 2×2=4
   (i) Almost sure convergence (ii) Convergence in distribution
   (b) Let X be a discrete random variable with support $R_X = [0, 1]$ and the p.m.f is

$$P_X(x) = \begin{cases} \frac{1}{3}, & \text{if } x = 1 \\ \frac{2}{3}, & \text{if } x = 0 \\ 0, & \text{otherwise} \end{cases}$$

   Prove that $\{X_n\}$ coverges in probability to X i.e. $X_n \xrightarrow{P} X$ 6

(c) Examine whether WLLN holds good for the sequence $\{X_n\}$ of independent random variables where $P\left(X_n = \frac{1}{\sqrt{n}}\right) = \frac{2}{3}$, $P\left(X_n = \frac{-1}{\sqrt{n}}\right) = \frac{1}{3}$. 3

(d) State Liapunov central limit theorem. 1

UNIT–II

3. (a) Define order statistics. Derive the cumulative distribution function of r^{th} order statistics. 5

(b) Derive the distribution of range, i.e., $X_{(n)} - X_{(1)}$ drawn from a population with probability density function $f(x)$. 4

(c) Derive the sample median which is drawn from a population with p.d.f. $f(x)$, when sample median is odd or even. 5

4. (a) For joint probability density function of two order statistics, prove that $f_{rs}(x, y) = F_{rs}(X_{(r)}, X_{(s)})$. 6

(b) Derive the probability density function (p.d.f.) and cumulative distribution function (c.d.f.) of first order statistics, i.e., $f_1(x)$ and $F_1(x)$. 4

(c) If $X \sim U(0, \theta)$ and $(X_1, X_2, ..., X_n)$ be a random sample drawn from uniform distribution. Find the density function of the smallest and largest/highest order statistics. 4

UNIT–III

5. (a) A wrong decision about H_0 leads to: 1

(i) one kind of error

(ii) two kinds of error

(iii) three kinds of error

(iv) none of the above

(b) Define level of significance and P-value. 1+1=2

(c) In two large populations, there are 30 and 25 per cent respectively of blue-eyed people. Is this difference likely to be hidden in samples of 1200 and 900 respectively from the two populations? 5

(d) Prove that S^2 is an unbiased estimate of the population variance σ^2. 6

6. (a) Testing of hypothesis $H_0 : \mu = 10$ vs $H_1 : \mu > 10$ leads to 1
   (i) one-sided left-tailed test
   (ii) one-sided right-tailed test
   (iii) two-tailed test
   (iv) all the above

   (b) Define one-tailed and two-tailed test. 1½+1½=3

   (c) A cigarette manufacturing firm claims that brand A of its cigarettes outsells its brand B by 8%. If it is found that 42 out of a sample of 200 smokers prefer brand A and 18 out of another random sample of 100 smokers prefer brand B, test whether the 8% difference is a valid claim. (Use 5% level of significance). 6

   (d) Describe the test of significance for the difference of standard deviation. 4

UNIT–IV

7. (a) Define chi-square distribution. Derive its distribution by the method of MGF. 2+8=10

   (b) Discuss the properties of chi-square distribution. 4

8. (a) Discuss the nature of p.d.f. curve for different degrees of freedom. 6

   (b) Obtain the m.g.f. of chi-square distribution and estimate the mean and variance. 8

UNIT–V

9. (a) Give the definition of Fisher's t-distribution and obtain its distribution. 2+6=8

   (b) Obtain the graph of t-distribution. 6

10. (a) If $\rho = 0$, then show that $t = \frac{r}{\sqrt{1-r^2}}\sqrt{(n-2)}$ is distributed as student's-t with $(n-2)$ d.f. 7

    (b) Obtain the mode and points of inflexion of F distribution. 4

(c) In $F(n_1, n_2)$ distribution, if we let $n_2 \to \infty$, then show that $\chi^2 = n_1 F$ follows χ^2-distribution with n_1 d.f. 3