2023
B.A./B.Sc.
First Semester
CORE – 2
STATISTICS
Course Code: STC 1.21
(Calculus)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) Discuss the kind of discontinuity, if any, of the function

$$f(x)=\begin{cases}\dfrac{x-|x|}{x} & ,if\ x\neq 0\\ 2 & ,if\ x=0\end{cases}$$ 4

(b) Find the n^{th} differential coefficient of $\log(1+x)$. 4

(c) Define partial differentiation. If $U=x^2y+y^2z+z^2x$ then prove that

$$\frac{\delta u}{\delta x}+\frac{\delta u}{\delta y}+\frac{\delta u}{\delta z}=(x+y+z)^2$$ 2+4=6

2. (a) Examine the following function for continuity at the origin

$$f(x)=\begin{cases}\dfrac{xe^{\frac{1}{x}}}{1+e^{\frac{1}{x}}} ,if\ x\neq 0\\ \quad ,if\ x=0\\ 0\end{cases}$$ 4

(b) If $U=f(y-z,z-x,x-y)$. Prove that $\dfrac{\delta u}{\delta x}+\dfrac{\delta u}{\delta y}+\dfrac{\delta u}{\delta z}=0$. 4

(c) Evaluate $\lim\limits_{n\to\infty}\dfrac{e^{5x}-1}{x}$ by L'Hospital's rule. 4

(d) State Leibnitz theorem. 2

UNIT–II

3. (a) Find the maxima and minima of the following functions 2+2=4

   (i) $4x^3 + 15x^2 + 12x - 2$

   (ii) $x^3 + y^2 + 6x + 12$

   (b) Derive the theorem for finding maxima and minima of a function by the method of Lagrange's multiplier. 6

   (c) By using Lagrange's multiplier find the maxima and minima values of $f(x, y) = 3x + 4y$ subject to constrains $x^2 + y^2 = 10$. 4

4. (a) Find the maxima and minima of the following functions: 2+3=5

   (i) $f(x) = x^3 - 9x^2 + 15x - 3$

   (ii) $f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$

   (b) If $U_1 = \frac{x_2 x_3}{x_1}, U_2 = \frac{x_1 x_3}{x_2}, U_2 = \frac{x_1 x_2}{x_3}$. Prove that $J(U_1, U_2, U_3) = 4$. 4

   (c) If $u = r\cos\theta, y = r\sin\theta$. Find $\frac{\delta(x, y)}{\delta(r, \theta)}$. 2

   (d) If $U = x + 3y^2 - z^3$, $V = 4x^2yz$, $W = 2z^2 - xy$. Evaluate

   $\frac{\delta(U,V,W)}{\delta(x, y, z)}$ at (1, –1, 0). 3

UNIT–III

5. (a) Define definite integral. Also, evaluate 1+1+1=3

   (i) $\int_4^6 3x^2 dx$

   (ii) $\int_2^4 6x^2 - 3x + 11 dx$

   (b) Define double and multiple integration. Also evaluate the following integrals. $\iint_R (x^2 + y^2) dxdy$; where R has a range

   $1 \le x \le 0, 1 \le y \le 0$. 2+3=5

(c) Evaluate $\int_0^a \int_y^a \frac{x}{x^2+y^2} dxdy$ by changing the order of integration. 6

6. (a) Define differential under integral sign. Also prove that

$$\int_0^1 \frac{x^{\alpha-1}}{\log x} dx = \log(1+\alpha); \alpha \geq 0.$$ 2+5=7

(b) Define beta and gamma function. Also prove that $\beta(m,n) = \beta(n,m)$.
2+2=4

(c) Prove that $\beta(m,n) = \frac{\Gamma m \sqrt{n}}{\sqrt{m+n}}$. 3

UNIT–IV

7. (a) Form the differential equation from the equation $u = \frac{a}{r} + b$, where u is dependent and r is independent variable. 3

(b) Solve: $(1-x)\, dy - (3+y)\, dx = 0.$ 3

(c) Solve the homogenous differential equation:
$x^2 y dx - (x^3 + y^3)\, dy = 0.$ 3

(d) Solve: $(D^2 - 4D + 1)y = cos2x.$ 5

8. (a) Find the differential equation of the family of curves $y = Ae^x + \frac{B}{e^x}$.
3

(b) Solve: $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0.$ 3

(c) Solve the homogeneous differential equation:
$xdy - ydx = \sqrt{x^2 + y^2}\, dx$. 4

(d) Solve the equation solvable for y: $y = 2px - p^2$. 4

UNIT–V

9. (a) Form the partial differential equation by eliminating m, n from

$$z = mxe^y + \frac{1}{2} m^2 e^{2y} + n.$$ 3

(b) Solve the Clairaut's partial differential equation
$z = px + qy + p^2 + q^2$. 4

(c) Solve by Charpit's method: $2xz - px^2 - 2qxy + pq = 0$. 4

(d) Classify the following operator: $\frac{\delta^2 u}{\delta x^2} + t\frac{\delta^2 u}{\delta x \delta t} + x\frac{\delta^2 u}{\delta t^2}$. 3

10. (a) Form the partial differential equation by eliminating a, b from
$z = mx + ny + m^2 + n^2$. 3

(b) Solve the equation $p(1 + q^2) = q(z - c)$ involving p, q and z. 4

(c) Solve the complete integral: $(p^2 + q^2)y = qz$. 4

(d) Classify the following operators: 1½×2=3

(i) $\frac{\delta^2 u}{\delta x^2} - 4\frac{\delta^2 u}{\delta x \delta y} + 2\frac{\delta^2 u}{\delta y^2}$

(ii) $\frac{\delta^2 u}{\delta x^2} + 4\frac{\delta^2 u}{\delta x \delta y} + 4\frac{\delta^2 u}{\delta y^2}$