**2023**
**B.A./B.Sc.**
**Third Semester**
CORE – 5
**PHYSICS**
*Course Code: PHC 3.11*
(Mathematical Physics - II)

*Total Mark: 70* *Pass Mark: 28*
*Time: 3 hours*

*Answer five questions, taking one from each unit.*

## UNIT–I

1. (a) Find the Fourier series of the function

$$f(x)=\begin{cases}-1 \text{ for } -\pi<x<\dfrac{-\pi}{2}\\ 0 \text{ for } \dfrac{-\pi}{2}<x<\dfrac{\pi}{2}\\ +1 \text{ for } \dfrac{\pi}{2}<x<\pi\end{cases}$$ 8

(b) Find the Fourier sine series for the function $f(x)=e^{ax}$ for $0<x<\pi$, where $a$ is a constant. 6

2. (a) If $f(x)=\begin{cases}\pi x, & 0<x<1\\ \pi(2-x), & 1<x<2\end{cases}$ using half range cosine series, show that $\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\ldots=\frac{\pi}{96}$. 8

(b) Find the complex form of the Fourier series of $f(x)=e^{-x}, -1\le x\le 1$. 6

## UNIT–II

3. (a) Find regular singular points of the differential equation:

$x(x-2)^2+2(x-2)y'+(x+3)y=0$ 4

(b) Solve the following equation in power series about $x=0$.

$$2x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-(x+1)y=0$$ 10

4. (a) Derive orthogonality of Legendre polynomials

$$\int_{-1}^{+1}p_m(x).p_n(x)dx=0; n\neq m$$ 8

(b) Prove that 6

(i) $p'_{n+1}-p'_{n-1}=(2n+1)p_n$

(ii) $p'_n-xp'_{n-1}=np_{n-1}$

## UNIT–III

5. (a) Prove that $J''_n=\frac{1}{4}[J_{n-2}(x)-J_{n+2}(x)]$. 4

(b) Show that $\int_{-\infty}^{\infty}e^{-x^2}H_m(x)H_n(x)=0$; for $n\neq m$.

$=2^n n!\sqrt{\pi};\ m=n$. 10

6. (a) Derive the following recurrence relation $L'_n(x)-nL'_{n-1}(x)+nL_n=0$. 4

(b) Use ascending power series method for the equation $x^2y''+xy'+(x^2-n^2)y=0$, to derive Bessel's function of first kind. 10

## UNIT–IV

7. (a) Write a short note on propagation error in addition and multiplication. 4

(b) Prove that gamma (1/2) $=\sqrt{\pi}$. 4

(c) Evaluate the integral $\int_0^1\left(\frac{x^3}{1-x^3}\right)^{1/2}dx$. 6

8. (a) Calculate the standard error for the data 2, 7, 9, 13, 16. 4

(b) Transform $\beta(l,m)$ to $\int_0^{\infty}\frac{x^{l-1}}{(1+x)^{m+n}}dx$. 4

(c) Evaluate $\int_0^{\infty} \frac{x^a}{(a)^x} dx$ and hence find $\int_0^{\infty} \frac{x^7}{(7)^x}$ . 6

## UNIT–V

9. Consider the heat flow through a thin bar of uniform cross-section of length $L$ and thermal conductivity $\sigma$ . One dimensional heat conduction equation is given by $\frac{\partial^2\theta}{\partial x^2} = \frac{1}{\sigma}\frac{\partial\theta}{\partial t}$ , where $\theta$ is the temperature. Solve the above equation with given boundary condition
$\theta(0,t) = \theta(l,0) = 0, t > 0$ and initial condition $\theta(x,t) = x,\ 0 < x < l$ . 14

10. (a) Using method of separation of variable, find the general solution of Laplace equation in rectangular coordinate system. 6

(b) Write the equation of vibration of tightly stretched rectangular membrane and obtain its general solution. 8