2023
B.A./B.Sc.
First Semester
CORE – 1
PHYSICS
Course Code: PHC 1.11
(Mathematical Physics)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) Solve the differential equation $x^4 + \frac{dy}{dx} + x^3 y = -\sec(xy)$ 3

(b) Define homogeneous differential equation and solve the differential equation $\frac{dy}{dx} = \frac{y}{x} + x\sin\frac{y}{x}$ 7

(c) Show that the given equation is an exact differential equation and hence solve it
$\{2xy \cos x^2 - 2xy + 1\}dx + \{\sin x^2 - x^2 + 3\}dy = 0$ 4

2. (a) If $y_1(x)$ and $y_2(x)$ are two linearly independent solutions of the homogeneous differential equation, $y'' + Py' + Qy = 0$, then show that the general solution is given by $y = Ay_1 + By_2$, where A and B are arbitrary constants. 4

(b) Solve 5×2=10

(i) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$, $y = 2$ and $\frac{dy}{dx} = \frac{d^2y}{dx^2}$ when $x = 0$.

(ii) $\frac{d^2y}{dx^2} + 4y = 2\cos x \cos 3x$

UNIT –II

3. (a) Four points having position vector $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are coplanar, show that

$$\left[\vec{a}\,\vec{b}\,\vec{c}\right] = \left[\vec{a}\,\vec{b}\,\vec{d}\right] + \left[\vec{a}\,\vec{d}\,\vec{c}\right] + \left[\vec{d}\,\vec{b}\,\vec{c}\right]$$ 3

(b) Prove that $\hat{i} \times (\hat{a} \times \hat{i}) + \hat{j} \times (\hat{a} \times \hat{j}) + \hat{k} \times (\hat{a} \times \hat{k}) = 2\hat{a}$. 5

(c) Find the angle between the surface $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at $(2, -1, 2)$. 6

4. (a) If $u = x + y + z$, $v = x^2 + y^2 + z^2$, $w = yz + zx + xy$, prove that grad u, grad v and grad w are coplanar vectors. 5

(b) Find the direction in which the directional derivative of $\phi(x, y) = \dfrac{x^2 + y^2}{xy}$ at (1,1) is zero and hence find the component of velocity of the vector $\overline{r} = (t^3 + 1)\hat{i} + t^2\hat{j}$ in the same direction at $t = 1$. 5

(c) Show that $\vec{A} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational. Find ϕ, such that $\vec{A} = \vec{\nabla}\phi$ 4

UNIT–III

5. (a) If $\vec{F} = 2y\hat{i} - z\hat{j} + x\hat{k}$, evaluate $\int_c \vec{F} \times \overrightarrow{dr}$ along the curve $x = \cos t$, $y = \sin t$, $z = 2\cos t$ from $t = 0$ to $t = \dfrac{\pi}{2}$. 5

(b) Evaluate $\iint_S (yz\hat{i} + zx\hat{j} + xy\hat{k}).\overrightarrow{ds}$, where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ in the first octant. 4

(c) State Green's theorem. Using Green's theorem, evaluate $\int_c (x^2 y\,dx + x^2 dy)$, where c is the boundary described counter clockwise of the triangle with vertices (0,0), (1,0), (1,1). 1+4=5

6. (a) Apply divergence theorem to evaluate $\int\int\int_V \vec{F}.\hat{n}\, ds$, where $\vec{F} = 4x^3\hat{i} - x^2y\hat{j} + x^2z\hat{k}$ and S is the surface of the cylinder $x^2 + y^2 = a^2$ bounded by the planes $z = 0$ and $z = b$. 6

(b) Verify Stoke's theorem for a vector field defined by $\vec{F} = \left(x^2 - y^2\right)\hat{i} + 2xy\hat{j}$ in the rectangle in xy-plane bounded by lines $x = 0, x = a, y = 0, y = b$. 8

UNIT–IV

7. (a) Derive the expression of curl $\overline{f}$ in the orthogonal curvilinear co-ordinates. 8

(b) Show that the spherical co-ordinate system $\left(\overline{T}_r, \overline{T}_\theta, \overline{T}_\phi\right)$ is self-reciprocal. 4

(c) Transform the wave equation $\frac{\partial^2 u}{\partial r^2} = c^2\nabla^2 u$ in spherical co-ordinates if u is independent of ϕ. 2

8. (a) If $x = uv\cos w,\ y = uv\sin w, z = \frac{1}{2}\left(u^2 - v^2\right)$; find h_1, h_2, h_3 and show that $ds^2 = \left(u^2 + v^2\right)\left(du^2 + dv^2\right) + uv\,dw^2$. 7

(b) For spherical co-ordinates, $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$, 7

(i) find the components of $\frac{\partial r}{\partial r}, \frac{\partial r}{\partial \theta}, \frac{\partial r}{\partial \phi}$

(ii) verify the mutual orthogonality of $\frac{\partial r}{\partial r}, \frac{\partial r}{\partial \theta}, \frac{\partial r}{\partial \phi}$

UNIT–V

9. (a) What are random variables? Explain with the help of an example. 1+2=3

(b) If a coin is thrown three times such that X is the number of heads. What are the expected values and variance of X ? 4

(c) A bag contains 3 black and 4 white balls. Two balls are drawn at random, one at a time without replacement.

(i) What is the probability that the second ball selected is white?

(ii) What is the conditional probability that the first ball selected is white if the second ball is known to be white? 5+2=7

10. (a) Prove that $\delta\left(x^2-a^2\right)=\dfrac{\delta(x-a)+\delta(x+a)}{2|a|}$ 5

(b) Evaluate the following: 3×2=6

(i) $\int_{-\infty}^{+\infty} \sin 2t \left(t-\frac{\pi}{4}\right) dt$

(ii) $\int_{-\infty}^{+\infty} e^{-5t}\delta(t-2)\,dt$

(c) Show that the Gaussian function, $G_\sigma(x)=\dfrac{1}{\sigma\sqrt{2\pi}}\exp\left[\dfrac{-x^2}{2\sigma^2}\right]$, $\sigma>0$

has a unit area under its curve. 3