2023
B.A./B.Sc.
Third Semester
SKILL ENHANCEMENT COURSE – 1

MATHEMATICS
Course Code: MAS 3.11

(Logic & Sets)

Total Mark: 35 *Pass Mark: 14*

Time: 2 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) Let p and q be the propositions: 1+1=2

p : I bought a lottery ticket this week

q : I won the million dollar jackpot

Express each of these propositions as an english sentence.

(i) $p \to q$

(ii) $\neg p \wedge \neg q$

(b) For each of these sentences, determine whether "an inclusive or", or "an exclusive or", is intended. Explain your answer. 2½+2½=5

(i) To enter the country you need a passport or a voter registration card.

(ii) A password must have at least three digits or be at least eight characters long.

2. (a) Which of the given sentences are propositions? What are the truth values of those that are propositions? 3

(i) $x + 1 = 4$

(ii) Kohima is the capital of Assam

(b) Suppose the value of $p \to q$ is T, what can be said about the value of $\neg p \wedge q \leftrightarrow p \vee q$. 4

UNIT–II

3. (a) Use De Morgan's laws to find the negation of each of the following:
   (i) John is rich and happy — 2×2=4
   (ii) Carlos will ride a bicycle or run tomorrow
   (b) Use truth tables to verify the absorption laws: — 3
   $p \vee (p \wedge q) \equiv p$ and $p \wedge (p \vee q) \equiv p$

4. (a) Let $F(x, y)$ be the statement "x can fool y", where the domain consists of all people in the world. Use quantifiers to express each of these statements. — 2+2=4
   (i) Everybody can fool Fred
   (ii) Everyone can be fooled by somebody
   (b) Determine the truth value of the statement $\exists n$ such that $n = -n$ if the domain is the set of all integers. — 3

UNIT–III

5. (a) State true or false with proper reasons: — 1+1=2
   (i) The sets $\{\{1, 2\}\}$ and $\{1, 2\}$ are equal
   (ii) $\{x\} \in \{x\}$
   (b) Show that if A and B are sets with $A \subset B$ then $|A| \leq |B|$. — 2
   (c) With the aid of a Venn diagram investigate the validity of the inference:
   If A, B and C are subsets of U such that $A \cap B \subseteq \overline{C}$ and $A \cup C \subseteq B$, then $A \cap C = \emptyset$. — 3

6. (a) For any subsets A, B and C of U, prove that — 4
   $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
   (b) How many license plates can be made by using either two uppercase English letters followed by four digits or two digits followed by four uppercase letters? — 3

UNIT–IV

7. (a) Prove that two sets are equal if and only if their symmetric difference is Ø. — 4

(b) Prove that for all sets A, B and C: 3

$(A - B) - C = (A - C) - (B - C)$

8. (a) For any two sets A and B, prove using set identities that 4

$A \subseteq B \Leftrightarrow A \cap B = A$ and $A \subseteq B \Leftrightarrow A \cup B = B$.

(b) For finite sets A and B, if $|A| = |B|$, then show that $|\mathcal{P}(A)| = |\mathcal{P}(B)|$. 3

UNIT–V

9. (a) If $A = \{a, b, c\}$ and $B = \{0, 1\}$. Find $B \times A$. What are the domain and range of this relation? 3

(b) Let R be a relation on the set of integers such that aRb if and only if $a = b$ or $a = -b$. Prove that R is an equivalence relation. 4

10. (a) Let R be an equivalence relation on a set A. Prove that for $a, b \in A$, the following statements are equivalent:

(i) aRb

(ii) $[a] = [b]$

(iii) $[a] \cap [b] \neq \emptyset$

where $[a]$ and $[b]$ are the equivalence classes of a and b respectively. 5

(b) Let R be a relation from $\mathbb{R}$ to $\mathbb{R}$ defined by $R = \{(a, b) \mid b \geq 0$ and $b \leq a$ and $a + b \leq 1\}$. Write this relation as an intersection of three relations. 2