2023

B.A./B.Sc.

First Semester

GENERIC ELECTIVE – 1

MATHEMATICS

Course Code: MAG 1.11

(Calculus)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) A point moves in a line so that its distance S cm measured from a fixed point O on the line at time t seconds reckoned from some fixed epoch is given by $S = t^3 - 6t^2 - 15t$. Find the following: 5
   (i) Velocity and acceleration at any instant, at the end of first second.
   (ii) The average velocity while t changes from 1 to 6 seconds.
   (iii) When and where the body stops.

   (b) If $y = a\cos(\log x) + b\sin(\log x)$ for $x > 0$, show that 5
   (i) $x^2y_2 + xy_1 + y = 0$
   (ii) $x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$

   (c) Find the n^{th} derivative of $y = \dfrac{x}{(x-1)(x-2)}$, using partial fraction method. 4

2. (a) If $y = \sin(\sin x)$, prove that $\dfrac{d^2y}{dx^2} + \tan x\dfrac{dy}{dx} + y\cos^2 x = 0$. 4

   (b) Find the n^{th} derivative of $y = \cos 5x \cos 2x$. 5

   (c) If $y = \tan^{-1}x$, show that 5
   (i) $(1+x^2)y_1 = 0$
   (ii) $(1+x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$

UNIT–II

3. (a) Verify Rolle's theorem in the interval $[a, b]$ for the function $f(x) = (x - a)^m (x - b)^n$; m, n being positive integers. 5

(b) Use mean value theorem to show that $\sqrt{101}$ lies between 10 and 10.5. 4

(c) Evaluate $\lim_{x\to 1}\left(\frac{x}{1-x} - \frac{1}{\log x}\right)$. 3

(d) Show that $\frac{\sin\theta}{\theta}$ continually decreases as θ increases from 0 to $\frac{\pi}{2}$. 2

4. (a) Prove by mean value theorem $1 + x < e^x < 1 + xe^x$, where $x \geq 0$. 4

(b) Show that $f(x) = -2x^3 + 15x^2 - 36x + 6$ is strictly increasing in $2 < x < 3$. 4

(c) Find the approximate value of $(80)^{\frac{1}{4}}$. 3

(d) Evaluate $\lim_{x\to a}\frac{\log(x-a)}{\log(e^x - e^a)}$. 3

UNIT–III

5. (a) Obtain the expansion of the function with the remainder of Lagrange's form

$$(1+x)^{1/5} = 1 + \frac{1}{5}x - \frac{2}{25}x^2 + \frac{6}{125}x^3 - \frac{42}{1250}x^4(1+\theta x)^{-19/5}.$$ 4

(b) Evaluate $\lim_{x\to 0}\frac{\sin x - \tan x}{x^3}$ using Taylor's series. 4

(c) Prove by repeated differentiation of the identity $(1 - x)^{-1} = 1 + x + x^2 + x^3 + \ldots$ where $|x| < 1$, that if m be a positive integer then

$$(1-x)^{-m} = 1 + mx + \frac{m(m+1)}{1.2}x^2 + \frac{m(m+1)(m+2)}{1.2.3}x^3 + \ldots$$ 6

6. (a) Expand $\sin x$ in powers of $\left(x - \frac{\pi}{2}\right)$ by Taylor's theorem. 5

(b) Evaluate $\lim\limits_{x\to 0}\dfrac{1-e^x}{1+x-e^x}$ using Taylor's series. 4

(c) Show that $\dfrac{1}{x}=\dfrac{1}{2}-\dfrac{1}{2^2}(x-2)+\dfrac{1}{2^3}(x-2)^2-\ldots;\ 0<x<4.$ 5

UNIT–IV

7. (a) Evaluate $\displaystyle\int\frac{x^2dx}{(x^2+a^2)(x^2+b^2)}$. 5

(b) Obtain reduction formula for $\int\cos^n xdx$; n being positive integer greater than 1 and hence evaluate $\int\cos^5 xdx$. 5

(c) Apply beta and gamma functions to prove that

$$\int_0^{\pi/2}\sin^4 x\cos^4 xdx=\frac{3\pi}{256}.$$ 4

8. (a) Evaluate $\displaystyle\int_0^{\pi/2}\frac{\sin x}{\sin x+\cos x}dx=\frac{\pi}{4}$. 5

(b) Prove that beta function, $B(m,n)=\displaystyle\int_0^1\frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx$. 5

(c) Show that $\displaystyle\int\frac{\sin^5 x}{\cos^4 x}dx=\frac{1}{3\cos^3 x}-\frac{2}{\cos x}-\cos x$. 4

UNIT–V

9. (a) Find the area of the surface generated by revolving about y-axis the part of the astroid $x=a\cos^3\theta$, $y=a\sin^3\theta$, that lies in the first quadrant. 5

(b) Given that the area between the curves $y^2=4ax$ and $x^2=4ay$, $(a>0)$, revolves about the x-axis. If V be the volume of the solid thus formed, then show that $V=\dfrac{96}{5}\pi a^2$. 5

(c) Find the perimeter of the cardioide $r=a(1+\cos\theta)$. 4

10. (a) Use method of rings to find the volume of the solid region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ when it is revolved about the x-axis to generate a solid. 5

(b) Find the area of the surface generated by revolving the portion of the curve $y = x^3$ between $x = 0$ and $x = 1$ about the x-axis. 5

(c) Find the length of the arc of the curve $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$ from $\theta = 0$ to $\theta = \pi$. 4
