2023

B.A./B.Sc.

Fifth Semester

DISCIPLINE SPECIFIC ELECTIVE – 2

MATHEMATICS

Course Code: MAD 5.21

(Boolean Algebra & Automata Theory)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) Define ordered set. Show that $(\mathbb{N}, \geq)$ is a totally ordered set. 1+3=4

   (b) Let P be an ordered set and $x, y \in P$. Then show that the following statements are equivalent 5

   (i) $x \leq y$

   (ii) $\downarrow x \subseteq \downarrow y$

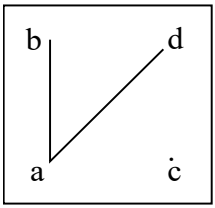
   (iii) $\forall Q \in \mathcal{O}(P)$ if $y \in Q \Rightarrow x \in Q$

   (c) Define order preserving and order embedding map between ordered sets with examples.

   Let $\varphi : P \to Q$ and $\psi : Q \to R$ be order preserving maps then show that the composite map

   $\psi \circ \varphi$ given by $(\psi \circ \varphi)x = \psi(\varphi(x))$ for $x \in P$ is also order preserving map. 2+3=5

2. (a) Draw and label a diagram of the ordered sets $\mathcal{O}(P)$ of down sets for the ordered set P given by the diagram 6



(b) Let P be a lattice, then prove that for all $a, b, c, d \in P$ 5

(i) $a \le b \Rightarrow a \vee c \le b \vee c$ and $a \wedge c \le b \wedge c$

(ii) $a \le b$ and $c \le d \Rightarrow a \vee c \le b \vee d$ and $a \wedge c \le b \wedge d$

(c) Define sublattice. Give one example to show that the subset of a lattice L is a lattice on its own but not a sublattice of L. 1+2=3

UNIT–II

3. (a) Prove that a lattice L is distributive if and only if the cancelation rule holds. 5

(b) Show that (B, gcd, lcm) is a Boolean algebra if B is the set of all positive divisor of 110. 4

(c) In a Boolean algebra B, show that $\forall x, y \in B$ 5

$$x \le y \Leftrightarrow x' \ge y' \Leftrightarrow x \wedge y' = 0 \Leftrightarrow x' \vee y = 1 \Leftrightarrow x \wedge y = x \Leftrightarrow x \vee y = y$$

4. (a) Using Quine-McCluskey method minimize the Boolean polynomial $p = \sum (0, 5, 8, 9, 10, 11, 14, 15)$ 5

(b) Find the disjunctive normal form of 2+2=4

(i) $x(y+z)' + (xy+z')x$

(ii) $((y+xz)(x+z)y)'$

(c) A motor is supplied by three generators where operation of each generator is monitored. Design a switching circuit to obtain the outputs satisfying the following conditions: 5

(i) A warning lamp lights up if one or two generator fails

(ii) An acoustic alarm is initiated if two or all three generators fails

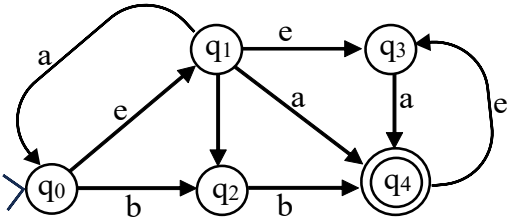
UNIT–III

5. (a) Find the regular expression and construct the finite automaton for the formal language 2+2=4

(i) $L = \{w \in \{a,b\}^* : ab \text{ is a substring of } w\}$

(ii) $L = \{w \in \{a,b\}^* : a \text{ and } b \text{ occurs even number of times in } w\}$

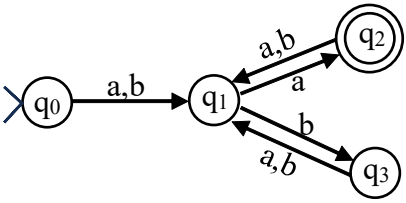
(b) Convert the given non-deterministic finite automata (NFA) to its equivalent deterministic finite automata (DFA). 7



(c) Design a non-deterministic finite automaton (NFA) that accepts strings over $\{a, b\}^*$ which contains a substring *aa* or *bb*. 3

6. (a) Show that regular language is closed under complementation and intersection. 2

(b) Using Arden's theorem find the regular expression for the language accepted by the deterministic finite automata (DFA) 6



(c) Show that $L = \{ww : w \in \{a, b\}^*\}$ is not a regular language. 6

UNIT–IV

7. (a) Define a regular context free grammar. Construct an NFA for the CFG given by: $V = \{S, A, B, a, b\}$; $\Sigma = \{a, b\}$;

$R = \{S \to bA; S \to aB; A \to abaS; B \to babS; S \to e\}$ 2+3=5

(b) Convert a grammar *G* to Chomsky's normal form, where the rules *R* of the grammar *G* is given by: $S \to aB \mid bA$; $A \to a \mid aS \mid bAA$;

$B \to b \mid bS \mid aBB$ 6

(c) Show that CFG $G = (V, \Sigma, R, S)$ where $V = \{S, a, b, +, *\}$, $\Sigma = \{a, b, +, *\}$, $S = S$, and

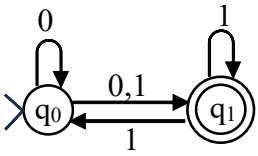
$R = \{S \to S + S; S \to S * S; S \to a; S \to b\}$ is an ambiguous grammar. 3

8. (a) Construct a PDA that accepts the language

$L = \{wcw^R : w \in \{a,b\}^*\}$. 5

(b) Prove that intersection of a CFL and a regular language is a CFL. 4

(c) Determine the equivalent PDA for the NFA given by the diagram: 5



UNIT–V

9. (a) Construct a Turing machine which compute the successor function. 5

(b) Do the machine LR and RL accomplishes the same thing? Discuss. 1+1=2

(c) Construct a Turing machine that compute the function

$f : \sum_0^* \to \sum_0^*$ define by $f(w) = ww^R$ and hence trace the input #*abb*#. 7

10. (a) Define a machine schema. Also, draw the copying machine and the right shifting standard machine. 1+4=5

(b) Differentiate between recursive language and recursively enumerable language. Prove that complement of a recursive language is recursive. 2+4=6

(c) Find a post correspondence solution for the given list M = (110, 0011, 0110) and N = (110110, 00, 110). 3