**2023**
**B.A./B.Sc.**
**Fifth Semester**
CORE – 11
**MATHEMATICS**
*Course Code: MAC 5.11*
(Multivariate Calculus)

*Total Mark: 70* *Pass Mark: 28*
*Time: 3 hours*

*Answer* two *questions, taking* one *from each unit.*

## UNIT–I

1. (a) Find the domain and range of $f(x,y)=\dfrac{1}{\sqrt{16-x^2-y^2}}$ also define limit of a function of two variables. 2+2+1=5

 (b) Evaluate $\lim\limits_{(x,y)\to(0,0)}\dfrac{3x^2y}{x^2+y^2}$ if it exists. 5

 (c) Investigate continuity at (0, 0) for the function

$$f(x,y)=\begin{cases}\dfrac{x^2-y^2}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$$ 4

2. (a) By using definition of partial derivatives $f(x,y)=4+2x-3y-xy^2$ evaluate $f_x(-2,1)$ and $f_y(-2,1)$. 2½+2½=5

 (b) If $z=e^2\sin y$ where $x=st^2$ and $y=s^2t$ find $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$ in terms of *s* and *t*. 2+2=4

 (c) Show that the function $w=\ln(2x+2ct)$ satisfies one-dimensional wave equation. 5

## UNIT–II

3. (a) Find the directional derivative of $f(x, y, z) = x^3 - xy^2 - z$ at $(1,1,0)$ in the direction $\vec{u} = 2\hat{i} - 3\hat{j} + 6\hat{k}$. In what direction does $f$ increase most rapidly at $(1,1,0)$. 3+2=5

(b) Find the equations of the tangent plane and normal line to the surface $\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 3$ at $(-2,1,-3)$. 2+2=4

(c) Find the absolute maximum and absolute minimum values of $f(x, y, z) = 2 + 2x + 4y - x^2 - y^2$ on the triangular region in the first quadrant bounded by the lines $x = 0, y = 0$ and $y = 9 - x$. 5

4. (a) Use Lagrange multipliers to find the extreme values of $f = xy$ subject to $4x^2 + 8y^2 = 16$. Also find the points. 4+1=5

(b) If $\phi$ is a scalar function and $\vec{F}$ is a vector field then show that $curl(\phi\vec{F}) = \phi curl\,\vec{F} + \vec{\nabla}\phi \times \vec{F}$. 4

(c) If $\vec{F} = x^2y\hat{i} + xz\hat{j} + 2yz\hat{k}$ then show that $div\, curl\, \vec{F} = 0$ 5

## UNIT–IIi

5. (a) Evaluate $\int_0^{\log 2}\int_1^{\log 5} e^{2x+y}\,dydx$. 4

(b) Evaluate $\int_0^1\int_0^1 \frac{x}{x^2 + y^2}\,dxdy$ 5

(c) Find the volume of the region bounded by the paraboloid $z = x^2 + y^2$ and below by the triangle enclosed by the lines $y = x$, $x = 0$, $x + y = 2$. 5

6. (a) Evaluate $\iint_R e^{x^2+y^2}\,dydx$ where $R$ is the semi-circular region bounded by $x$-axis and the curve $y = \sqrt{1 - x^2}$. 4

(b) Evaluate $\int_{0}^{2}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}}\int_{0}^{2x+y} dzdydx$ 5

(c) Use cylindrical co-ordinate system to find $\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_{0}^{9-x^2-y^2} x^2 dzdydx$. 5

## UNIT–IV

7. (a) By changing variables $x = u^2 - v^2, y = 2uv$ such that $0 \le u \le 1$, $0 \le v \le 1$, evaluate $\iint_R ydA$ where *R* is the region bounded by the *x*-axis and parabolas $y^2 = 4 - 4x$ and $y^2 = 4 + 4x$. 5

(b) Evaluate $\int_C 2xds$ where *C* consists of the arc $C_1$ of the parabola $y = x^2$ from (0, 0) to (1, 1) followed by the vertical line segments $C_2$ from (1, 1) to (1, 2). 5

(c) Find the work done by the force field $\vec{F}(x, y) = 3xy\hat{i} - 10x\hat{k}$ in moving a particle along the curve $x = t^2 + 1, y = 2t^2, z = t^3$ from $t = 1$ to $t = 2$. 4

8. (a) Evaluate $\int_C (yz + zx + xy)ds$ where *C* is the curve $\vec{r}(t) = a\cos t\hat{i} + b\sin t\hat{j} + ct\hat{k}$, $0 \le t \le \frac{\pi}{2}$. 4

(b) Show that the integral $\int_{(1,1)}^{(3,3)} \left\{\left(e^x \log y - \frac{e^y}{x}\right)dx + \left(\frac{e^x}{y} - e^y \log x\right)dy\right\}$ where *x*, *y* are positive, is independent of path and find its value by using fundemental theorem of integral calculus. 4+1=5

(c) Use the transformations $u = x, v = z–y, w = xy$ to calculate $\iiint_R (z - y)^2 xy\, dv$ where *R* is the region enclosed by the surfaces $x = 1, x = 3, z = y, z = y + 1, xy = 2, xy = 4$. 5

## UNIT–V

9. (a) Verify Green's theorem for the vector field $\vec{F} = -x^2 yi + xy^2 \hat{j}$ and bounded the region by the circle $\vec{r}(t) = a\cos t\hat{i} + a\sin t\hat{j}$, $0 \le t \le 2\pi$. 5

   (b) Evaluate $\iint_S \vec{F}.\vec{n}ds$ where $\vec{F} = 18z\hat{i} - 12\hat{j} + 3y\hat{k}$ and $S$ is the part of the plane $2x + 3y + 6z = 12$ in the first octant. 5

   (c) By using Stoke's theorem prove that $div\left(curl\,\vec{F}\right) = \vec{0}$ for a vector field $\vec{F}$. 4

10. (a) State divergence theorem. Use divergence theorem to evaluate $\iint_s (x+z)\,dydz + (y+z)dzdx + (x+y)\,dxdy$ where $S$ is the surface of the sphere $x^2 + y^2 + z^2 = 4$. 1+4=5

    (b) Verify Stoke's theorem for $\vec{F} = \left(x^2 + y^2\right)\hat{i} - 2xy\,\hat{j}$ taken round the rectangle bounded by $x = \pm a$, $y = 0$ and $y = b$. 5

    (c) If $S$ is any closed surface enclosing a volume $V$ and $\vec{F} = x\hat{i} + 2y\hat{j} + 3z\hat{k}$ then prove that $\iint_S \vec{F}.\vec{n}ds = 6V$ 4