2022
B.A./B.Sc.
Third Semester
GENERIC ELECTIVE – 3
STATISTICS
Course Code: STG 3.11
(Basics of Statistical Inference)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five *questions, taking* one *from each unit.*

UNIT–I

1. (a) Define the following terms: 2×3=6
   (i) Estimate
   (ii) Consistent estimate
   (iii) Unbiased estimate of a parameter

   (b) Show that $\frac{\left[\sum x_i \left(\sum x_i - 1\right)\right]}{n(n-1)}$ is an unbiased estimate of θ^2, for the sample $x_1, x_2, ..., x_n$ drawn on X which takes the values 1 and 0 with respective probabilities θ and $1-\theta$. 3

   (c) X_1, X_2 and X_3 is a random sample of size 3 from a population with mean μ and variance σ^2. T_1, T_2 and T_3 are the estimators used to estimate mean value μ, where $T_1 = X_1 + X_2 - X_3$,

   $T_2 = 2X_1 + 3X_3 - 4X_2$ and $T_3 = \frac{1}{3}\left(\lambda X_1 + X_2 + X_3\right)$.

   (i) Are T_1 and T_2 unbiased estimators?
   (ii) Find the value of λ such that T_3 is an unbiased estimator of μ.
   (iii) Which is the best estimator?

   1+2+2=5

2. (a) When would you say that the estimate of a parameter is good? Prove that if $T(X_1, X_2, ..X_n)$ be an unbiased estimator for θ, it does not necessarily mean that T^2 will be an unbiased estimator for θ^2. 2+3=5

(b) State the sufficiency of an estimator. Let $x_1, x_2, ..., x_n$ be a random sample from $N(\mu, \sigma^2)$ population. Find sufficient estimators for μ and σ^2. 1+3=4

(c) Explain the maximum likelihood estimator and write down its properties. 5

UNIT–II

3. (a) What are simple and composite statistical hypothesis. Give examples. 2+1=3

(b) Let p be the probability that a coin will fall head in a single toss in order to test $H_0 : p = \frac{1}{2}$ against $H_1 : p = \frac{3}{4}$. The coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtained. Find the probability of Type I error and power of test. 5

(c) State and prove Neyman-Pearson lemma. 6

4. (a) What is a statistical hypothesis? Define (i) Type I and type II error, (ii) Power of test, (iii) Level of significance, with reference to testing of a hypothesis. Explain how the best critical region is determined. 1+2+1+1+2=7

(b) Explain the concept of most powerful test. 2

(c) Let X has a p.d.f. of the form

$$f(x,\theta) = \begin{cases} \frac{1}{\theta} e^{\frac{-x}{\theta}}, & 0 < x < \infty,\ \theta > 0 \\ 0, & \text{otherwise} \end{cases}$$

to test $H_0 : \theta = 2$ against $H_1 : \theta = 1$, use the random sample (x_1, x_2) of size 2 and define a critical region $W = \{(x_1, x_2) : 9.5 \le x_1 + x_2\}$. Find the (i) power of the test, (ii) significance level of the test. 5

UNIT–III

5. (a) Describe *t*-test for difference of means. 7
   (b) Explain χ^2 test for goodness of fit and Yates's correction.
   3½+3½=7

6. (a) Briefly explain the test of significance for paired *t*-test and *z*-test for single mean. 7
   (b) Discuss *F*-test of variance and test of significance of correlation coefficient. 7

UNIT–IV

7. (a) Write the correct answer: 1×2=2
   (i) Simple random sampling is a
   (A) probability sampling (B) non-probability sampling
   (C) mixed sampling (D) purposive sampling
   (ii) If from a population of size 6, a sample of size 3 is drawn without replacement, then the total number of possible samples will be
   (A) 18 (B) 15
   (C) 20 (D) 21
   (b) Write a note on sampling and non-sampling errors. 6
   (c) Show that in simple random sampling without replacement (SRSWOR) the sample mean square is an unbiased estimate of the population mean square. 6

8. (a) Write the correct answer: 1×2=2
   (i) Sampling error arises due to the use of
   (A) purposive sampling (B) sampling techniques
   (C) probability sampling (D) census survey
   (ii) A sample consists of
   (A) all units of the population
   (B) 50% units of the population
   (C) 25% units of the population
   (D) a representative part of the population
   (b) Explain about the basics principles of sample survey. 6

(c) Distinguished between simple random sampling with replacement (SRSWR) and simple random sampling without replacement(SRSWOR). Show that in SRSWOR the variance of the estimate of the population is given by $V(\bar{x}_n) = (1-f)\frac{S^2}{n}$. 2+4=6

UNIT–V

9. (a) What is analysis of variance technique? Mention the assumptions for the validity of F-test. 3+2=5

(b) What are the basic principles of design of experiment? Explain each of them elaborately. 7

(c) Obtain the missing values of the following ANOVA table of a completely randomized design. 4×½=2

Sources of Variation	d.f.	SS	MS	F
Treatment	4	--	--	2.5
Error	--	--	20	
Total	9	500		

10. (a) Represent the yield of a plot of a one-way classified data by a fixed effect additive linear model and estimate the parameters involved in it. 2+4=6

(b) Give an outline of statistical analysis of a randomized block design. 8