2022

B.A./B.Sc.

Fifth Semester

CORE – 11

STATISTICS

Course Code: STC 5.11

(Satochastic Processes & Queueing Theory)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five *questions, taking* one *from each unit.*

UNIT–I

1. (a) Obtain the probability generating function of Poisson distribution and hence obtain its mean and standard deviation. 3+4=7

(b) If $P(s)$ is the p.g.f. of a random variable X, find the p.g.f. of $\frac{X-a}{b}$, where a and b are any arbitrary constants. 3

(c) Find the p.g.f. of a random variable X for which $P(X \leq n)$ 4

2. (a) If $X_i, (i=1,2,...,n)$ are independent random variables, then obtain the probability generating function of $Z = \sum_{i=1}^{n} X_i$ in usual notations. 2

(b) Let X be a random variable with probability mass function $P(X=n) = q^{n-1}p$ for n=1,2,3,... Obtain the probability generating function of X and also $E(X)$ and $SD(X)$. 6

(c) Consider a series of Bernoulli trails with probability of success p. Suppose that X denotes the number of failures preceding the first success and Y denotes the number of failures following the first success and preceding the second success. The sum $X+Y$ gives the number of failures preceding the second success. Find the probability generating function of $X+Y$. 4

(d) Define hazard function and survival function. 2

UNIT–II

3. (a) Define the following: 6
Unit-step and m-step transition probability, transition probability matrix

(b) The transition probability matrix of a Markov chain $\{X_n, n \geq 0\}$

having three states 1, 2 and 3 is $p = \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$ and the initial

distribution is $P(X_0 = 1) = 0.7$, $P(X_0 = 2) = 0.2$,

$P(X_0 = 3) = 0.1$.

Find (i) $P(X_2 = 2, X_1 = 1, X_0 = 3)$

(ii) $P(X_2 = 3)$

(iii) $P(X_2 = 3 / X_0 = 2)$ 2+4+2=8

4. (a) Define Markov chain. When is a Markov chain homogenous? 3

(b) Three children (denoted by 1, 2 and 3) arranged in a circle play a game of throwing a ball to one another. At each stage, the child having the ball is equally likely to throw it to any one of the other two children. Suppose that X_0 denotes the child who had the ball initially and $X_n (n \geq 1)$ denotes the child who had the ball after *n* throws.

Show that $X_n (n \geq 1)$ forms a Markov chain. Find transition probability matrix. Also, calculate

(i) $P(X_2 = 3, X_1 = 2, X_0 = 1)$

(ii) $P(X_2 = 3)$

(iii) $P(X_2 = 2 / X_0 = 3)$ 2+2+2+2=8

(c) Define persistent, transient, and ergodic states. 3

UNIT–III

5. (a) Show that the sum of two independent Poisson process is a Poisson process. 5

   (b) Define and derive pure birth process with usual notations. 2+7=9

6. (a) Show that the interval between two successive occurrences of a Poisson process having parameter λ has a negative exponential distribution with mean $\frac{1}{\lambda}$ 6

   (b) What do you understand by pure death process. Show with usual notation that pure death process follows truncated Poisson distribution. 1+7=8

UNIT–IV

7. (a) Write briefly on the factors which completely describe the queueing system. 6

   (b) Prove that if the arrivals are completely random, then the probability distribution of the number of arrivals in a fixed time interval follows a Poisson distribution. 8

8. (a) Define transient state, steady state, and explosive state of a queueing system. What is traffic intensity? 6

   (b) State and prove the Markovian property of inter-arrival times. Mention four probabilistic and two deterministic queueing models. 5+3=8

UNIT–V

9. (a) What does the term "equilibrium price" mean? The supply and demand functions for three commodities X, Y, Z are given as:

$$d_x = 21 - 5p_x + 3p_y - 3p_z; s_x = 3 + p_x$$

$$d_y = 12 + 3p_x - 6p_y + 3p_z; s_y = 15 + 6p_y$$

$$d_z = 64 - 3p_x - 3p_y - 9p_z; s_z = 10 + 6p_z$$

Find the equilibrium prices and the equilibrium quantities exchanged for all the three commodities. 2+5=7

(b) What is Engel's law and Engel's curve? The price elasticity of demand curve $x = f(p)$ is of the form $(a - bp)$ where a and b are given constants. Find the demand curve. 4+3=7

10. (a) Define price elasticity of demand. Briefly explain what the different types of data required for estimating price elasticities of demand are. 2+6=8

(b) Describe Pareto's law of income distribution. 6