2022
B.A./B.Sc.
Third Semester
CORE – 7
STATISTICS
Course Code: STC 3.31
(Mathematical Analysis)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five *questions, taking* one *from each unit.*

UNIT–I

1. (a) Show that the given set is a bound set. Find the supremum and infimum of the set if they exists.

   (i) $S_n = \left\{ \frac{2n-3}{3n+4}, n \in N \right\}$

   (ii) $S_n = \left\{ -\left(\frac{n+1}{n} \right), n \in N \right\}$ 2+2=4

   (b) Define limit point of a set. 2

   (c) State and prove Bolzano-Weierstrass theorem. 6

   (d) Identify the following sequence and show whether it is monotonically increasing, monotonically decreasing, strictly increasing, or strictly monotonically decreasing.

   (i) $\left\langle -1, -\frac{1}{2}, -\frac{1}{3} \ldots\ldots\ldots \right\rangle$

   (ii) $\langle -2, 2, -4, 4, -6, 6, \ldots. \rangle$ 1+1=2

2. (a) Prove that the union of an arbitrary open sets is an open set. 4

   (b) Show whether the given sequence is convergent, divergent, or oscillatory? 2×2=4

   (i) $\langle a_n \rangle = \left\langle \frac{n^2+3}{n+2} \right\rangle$ (ii) $\langle a_n \rangle = \left\langle \frac{1}{n(n+1)}, n \in N \right\rangle$

(b) State and prove Cauchy's general principal of convergence of sequence. 6

UNIT–II

3. (a) State the Gauss' test. 3

(b) Show that the series $\sum\left(\frac{1}{n^p}\right)$ converges, if $p > 1$, and diverges if $p \leq 1$. 6

(c) Test for the convergence of the series

$$\sum \frac{n^2-1}{n^2+1} x^n, x > 0$$ 5

4. (a) State and prove Leibnitz test for the convergence of alternating series. 8

(b) Test the convergence of

$$\frac{1}{\sqrt{2-1}} - \frac{1}{\sqrt{3-1}} + \frac{1}{\sqrt{4-1}} - .....$$ 6

UNIT–III

5. (a) If $|f(y) - f(x)| \leq |y - x|^3 \; \forall x, y \in \mathbb{R}$. Find $f'(x)$. 3

(b) Define uniform continuous function. Show whether the function $f(x) = x^2, x \in (a,b), x, y \in \mathbb{R}$ and $|x - y| < \delta$, is uniformly continuous or not. 2+3=5

(c) Verify Rolle's theorem for $f(x) = x^2 - 5x + 6$ on [2,3]. 4

(d) Find the expansion of $\log(1+x)$. 2

6. (a) Examine the following function for continuity at the origin:

$$f(x) = \begin{Bmatrix} \dfrac{xe^{\frac{1}{x}}}{1+e^{\frac{1}{x}}}, & x \neq 0 \\ 0, & x = 0 \end{Bmatrix}$$ 3

(b) Define Lagrange's mean value theorem and show its geometrical interpretation. Verify Lagrange's value theorem for $f(x)=x^3-x^2-5x+3$ on [0,4] 3+4=7

(c) Apply Maclaurin's theorem and prove that

$$\log(\sec x)=\frac{1}{2}x^2+\frac{1}{12}x^4+\frac{1}{45}x^6+\ldots$$ 4

UNIT–V

7. (a) Show that $E=1+\Delta$. For any constant c, where is the value $\Delta(c)$ and $E(c)$? 2+2=4

(b) State and prove Newton-backward interpolation formula. 6

(c) Estimate the missing entries in the following table: 4

x:	1	2	3	4	5
$f(x)$:	2	___	7	___	32

8. (a) Evaluate: $\Delta^3(1-x)(1-2x)(1-3x)$, the interval of difference being unity. 3

(b) State and prove Gauss's forward central difference formula. 6

(c) State Lagrange's interpolation formula. Using this formula, find $f(4)$ from the following data: $f(1)=2, f(3)=8, f(7)=128$

2+3=5

UNIT–V

9. (a) Using the general quadratic formula, derive Simpson's $\frac{1}{3}$ rd rule formula for numerical integration and give some of the conditions for the validity if this rule. 5+2=7

(b) If $U_x=a+bx+cx^2$, prove that

$$\int_1^3 U_x dx = 2U_x+\frac{1}{12}(U_0-2U_2+U_4)$$ 3

(c) Prove Weddle's rule of numerical integration. 4

10. (a) Derive trapezoidal rule formula for numerical integration. In this rule, what is the degree of polynomial for $f(x)$? 5+1=6

(b) Evaluate $\log_e z$ using Weddle's rule with 7 ordinates to evaluate $\int_0^3 \frac{1}{1+x} dx$. 4

(c) Prove Simpson's $\frac{3}{8}$ th rule of numerical integration. 4