

**2022**  
**B.A./B.Sc.**  
**Third Semester**  
CORE – 5  
**STATISTICS**  
*Course Code: STC 3.11*  
(Sampling Distributions)

*Total Mark: 70*  
*Time: 3 hours*

*Pass Mark: 28*

*Answer five questions, taking one from each unit.*

**UNIT-I**

1. (a) State and prove weak law of large numbers. 7  
(b) State the Chebyshev's inequality. What are its uses? 4  
(c) A symmetric die is thrown 600 times. Find the lower bound for the probability of getting 80 to 120 sixes. 3
2. (a) Define the concepts: convergence in probability, almost sure convergence, convergence in mean square and convergence in distribution. 2×4=8  
(b) Let  $X_1, X_2, \dots, X_n$  be iid  $\text{Poi}(\lambda)$  variates. Let  $S_n = X_1 + X_2 + \dots + X_n$ . Using central limit theorem, show how to obtain an approximate value of  $P(a < S_n < b)$  using CLT. 6

**UNIT-II**

3. (a) Define the  $r^{\text{th}}$  order statistic and obtain its distribution. 2+6=8  
(b) Hence or otherwise obtain the distribution of the smallest and largest order statistic. 3+3=6
4. (a) Obtain the joint distribution of the  $r^{\text{th}}$  and  $s^{\text{th}}$  order statistic. Hence or otherwise, obtain the distribution of the range. 8

- (b) Let  $X_1, X_2, \dots, X_n$  be iid  $\exp(\lambda)$  variates. Find the distribution of the smallest order statistics i.e the distribution of
- $$Y_{(1)} = \min(X_1, X_2, \dots, X_n).$$
- 6

### UNIT-III

5. (a) Define the terms: Null hypothesis, composite hypothesis, type-I and type-II errors, critical region and p-value, power of a test.  $2 \times 7 = 14$
6. (a) Define how to test the significance of difference of  
 (i) two proportions and (ii) two means. 4+4=8  
 (b) Explain the standard error of sample mean. 3  
 (c) Distinguish between parameter and statistics giving examples. 3

### UNIT-IV

7. (a) Define a chi square variate with  $n$  d.f and derive its distribution. 2+6=8  
 (b) Show that variance of chi square distribution is twice its mean. 6
8. (a) If  $X$  and  $Y$  are chi square variates with  $m$  and  $n$  d.f. respectively, find the distribution of  $X/Y$ . 8  
 (b) State and prove the additive property of chi square distribution. 6

### UNIT-V

9. (a) Obtain the distribution of Fisher's  $t$ -distribution. 8  
 (b) Discuss the limiting form of  $t$ -distribution. 6
10. (a) Discuss the relationship between  $F$ -distribution and chi square distribution. 8  
 (b) If  $F$  follows  $F$ -distribution with  $(m, n)$  d.f, show that  $1/F$  follows  $F$  distribution with  $(m, n)$  d.f. 6