2022
B.A./B.Sc.
First Semester
CORE – 2
STATISTICS
Course Code: STC 1.21
(Calculus)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) If $f(x)=\dfrac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}$, does the $\lim\limits_{x\to 0} f(x)$ exists? Give reasons. 4

(b) Define total differentiation. If $f\left(\dfrac{x}{y},\dfrac{y}{z},\dfrac{z}{x}\right)$, then prove that

$$x\frac{du}{dx}+y\frac{du}{dy}+z\frac{du}{dz}=0$$ 2+4=6

(c) Find the n^{th} differential coefficient of $x^4\cos 3x$. 4

2. (a) If $f(x+y)=f(x)+f(y); x,y\in R$ and $f(x)=x^2g(x); g(x)$ is continuous, then find $f'(x)$. 3

(b) Evaluate $\lim\limits_{x\to\infty}\dfrac{11x+e^{-x}}{7x}$ by L' Hospital's rule. 3

(c) If $U=\tan^{-1}\left(\dfrac{x^3+y^3}{x-y}\right)$, then show that $x\dfrac{\partial U}{\partial x}+y\dfrac{\partial U}{\partial y}=\sin 2U$. 4

(d) Prove Euler's theorem for homogenous function. 4

UNIT–II

3. (a) Evaluate *any two* of the following: 2×2=4

(i) $\int_0^1 \int_0^\pi y \cos xy\, dx\, dy$ (ii) $\int_0^1 \int_0^{x^2} \int_0^{x+y} (2x - y - z)\, dz\, dy\, dx$

(iii) $\int_1^{\log 8} \int_0^{\log y} e^{x+y}\, dx\, dy$

(b) Prove that $\int_0^\infty x^{2n-1} e^{-ax^2} dx = \frac{\Gamma(n)}{2a^n}$ where $n > 0$ and $a > 0$. 5

(c) Change the order of integration and evaluate $\int_0^1 \int_{x^2}^{x} (2x - y - 3)\, dy\, dx$. 5

4. (a) Evaluate the following: 1½×2=3

(i) $\Gamma\left(\frac{-5}{2}\right)$ (ii) $B(3, 2.5)$

(b) Evaluate *any two* of the following: 2×2=4

(i) $\int_0^\infty x^2 e^{-5x^2} dx$ (ii) $\int_0^\pi \int_0^{\sin x} y\, dy\, dx$

(iii) $\int_0^\infty \frac{x^8\left(1 - x^6\right)}{\left(1 + x\right)^{24}} dx$

(c) Prove that $B(m+1, n) + B(m, n+1) = B(m, n)$. 2

(d) Transform to polar coordinates and integrate $\iint \sqrt{\frac{1 - x^2 - y^2}{1 + x^2 + y^2}}\, dx\, dy$, the integral being extended over all positive values of x and y subject to $x^2 + y^2 \le 1$. 5

UNIT–III

5. (a) If $a > b > 0$ and $f(\theta) = \frac{\left(a^2 - b^2\right)\cos\theta}{a - b\sin\theta}$, then find the maximum value of $f(\theta)$. 6

(b) Define Jacobian transformation. Find the Jacobian of $y_1, y_2, \ldots, y_n$; being given

$y_1 = x_1(1-x_2), y_2 = x_1x_2(1-x_3), \ldots, y_{n-1} = x_1x_2\ldots x_{n-1}(1-x_n),$

$y_n = x_1x_2\ldots x_n$ 5

(c) Describe the method of Lagrange's multiplier. 3

6. (a) Find the extreme values of the function $f(x,y) = x^2 + y^2$ subject to the constraint $x^2 - 2x + y^2 - 4y = 0$ 6

(b) If $u = x^2 + y^2 + z^2, v = x + y + z, w = xy + yz + zx$ then show that the Jacobian $J = \frac{\partial(u,v,w)}{\partial(x,y,z)}$ vanishes identically. 4

(c) Find the maximum value of the function $f(x) = \left(\frac{1}{x}\right)^x$ 4

UNIT–IV

7. (a) Find the differential equation corresponding to the family of curves $y = c(x-c)^2$ 2

(b) Solve: $y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$ 2

(c) Solve the differential equation reducible to homogeneous form:

$(2x - y + 1)dx + (2y - x - 1)dy = 0$ 2

(d) Solve the linear differential equation:

$x^2(x^2-1)\frac{dy}{dx} + x(x^2+1)y = x^2 - 1$ 4

(e) Solve the linear differential equation: $\frac{d^2y}{dx^2} + 9y = \sec 3x$ 4

8. (a) Find the differential equation of all the circles

$(x-h)^2 + (y-k)^2 = c^2$. 3

(b) Solve: $(1+x^2)dy + (1+y^2)dx = 0$ 2

(c) Solve the homogeneous differential equation:

$x^2ydx - (x^3+y)^3 dy = 0$ 3

(d) Solve the Bernoulli's equation: $x\frac{dy}{dx}+y=y^2\log x$ 3

(e) Solve the first order equation: $x^2p^2+3xyp+2y^2=0$ 3

UNIT–V

9. (a) Form the partial differential equation by eliminating a, b from

$z=axe^y+\frac{1}{2}a^2e^{2y}+b$ 3

(b) Solve by Lagrange's method: $(mz-ny)p+(nx-lz)q=(ly-mx)$ 3

(c) Solve by Charpit's method: $(p^2+q^2)y=qz$ 4

(d) Solve the linear equation: $\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial x\partial y}=\cos x\cos 2y$ 4

10. (a) Form the partial differential equation by eliminating a, b from

$z=ax+by+a^2+b^2$ 3

(b) Solve the nonlinear equation: $p(1+q^2)=q(z-a)$ 3

(c) Solve: $xq^2=px+qy+z$ 4

(d) Solve: $\frac{\partial^2 z}{\partial x^2}-2\frac{\partial^3 z}{\partial x^2\partial y}=2e^{2x}+3x^2y$ 4
