2022
B.A./B.Sc.
Fifth Semester
CORE – 11
PHYSICS
Course Code: PHC 5.11
(Quantum Mechanics & Applications)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five *questions, taking* one *from each unit.*

UNIT–I

1. (a) Define linear vector space. Explain the rule for vector addition and a rule for scalar multiplications. 6

(b) Let $U = \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$ and $V = \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}$, find their inner product and length of each. 4

(c) Find the eigenvalues and corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$$ 4

2. (a) Explain Gramm-Schmidt orthogonalization procedure. If Ψ and Φ be two vectors in inner product space, then prove the parallelogram law $\|\Psi + \Phi\|^2 + \|\Psi - \Phi\|^2 = 2\|\Psi\|^2 + 2\|\Phi\|^2$ 6

(b) Explain Hermitian matrix. Show that the matrix B*AB is Hermitian if A is Hermitian. 4

(c) Prove that the eigenvalues of unitary matrices are of unit modulus. 4

UNIT–II

3. (a) Explain Dirac notation with an example and give its properties. 4
   (b) What is Hermitian operator? Show that the momentum operator is a Hermitian operator. 5
   (c) Show that the commutator of position coordinates and momentum component which does not correspond to it is always zero. 5

4. (a) Prove that if a system of particle is in an eigenstate of one of the components, it cannot be in an eigenstate of either of the two other components. 5

   (b) If $H = \frac{p^2}{2m} + V(x)$, then show that $\left[x,\left[x,H\right]\right] = \frac{-\hbar^2}{m}$. 3

   (c) Show that the eigenvalue of L_z operator is $m\hbar$. 6

UNIT–III

5. (a) What is photoelectric effect? Discuss how classical physics fails to explain the photoelectric effect. How did Einstein theory overcome this difficulty? 6
   (b) Calculate the de Broglie wavelength of an electron travelling with velocity 3/5 c, where c is the speed of light. 2
   (c) What is de Broglie hypothesis? Show that the electron behaves like a wave based on the experiment conducted by Davisson and Germer. 6

6. (a) Explain the Rutherford atomic model based on the alpha-scattering experiment. Give its limitations. 6
   (b) What is Heisenberg uncertainty principle? Does it arise because of some error in the measurement technique? 2
   (b) Explain the different spectral series of H-atom and draw its energy level diagram. 6

UNIT–IV

7. (a) Normalize the wave function $\Psi(x) = xe^{-\alpha x^2}$ in the range $-\infty$ and $+\infty$ 4

(b) Obtain Schrödinger time dependent equation. 4

(c) Explain probability current and probability density. Derive an equation for probability conservation. 6

8. (a) Find the expectation value of position and momentum for a particle described by the wave function $\Psi(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi r}{L}$, for $0 < x < L$ 6

(b) Why should the wave function $\Psi(x)$ be single valued everywhere? 2

(c) Show that $\frac{d < P >}{dt} = \left\langle -\frac{V}{dt} \right\rangle$. 6

UNIT–V

9. (a) Obtain the solution for a particle in one-dimensional finite square well potential. 5

(b) Find the energy of the first excited state of an electron confined to move in a one dimension potential box of 1 Å.
[Given, $m = 9.11 \times 10^{-31}$ kg, $\hbar = 1.054 \times 10^{-34}$ Js] 2

(c) What is meant by barrier penetration? Deduce the expression for reflection and transmission coefficient for a step potential. 7

10. (a) What is tunneling effect? Show that there is always some probability of transmission through the potential barrier for $E < V_0$. 7

(b) The energy of a linear harmonic oscillator in the third excited state is 0.1 eV. Find the frequency of oscillation. 2

(c) Show that the probability of finding the particle outside the classical limits is approximately 16%, assuming that the oscillator is in its normal state. 5
