2022
B.A./B.Sc.
Third Semester
CORE – 5
PHYSICS
Course Code:PHC 3.11
(Mathematical Physics - II)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) What is a periodic function? Write the Dirichlet's conditions for a Fourier series. 1+5=6

(b) Find the Fourier series expansion for $f(x) = x + \frac{x^2}{4}, -\pi \le x \le \pi$. 8

2. (a) What are even and odd functions? Write the Fourier series expansions for an even and odd function. 2+4=6

(b) If $f(x) = \begin{cases} \pi x, & 0 < x < 1 \\ \pi(2-x), & 1 < x < 2 \end{cases}$, using half range cosine series, show

that, $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots = \frac{\pi^4}{96}$ 8

UNIT–II

3. (a) Using Frobenius method, obtain a series solution in powers of x for

differential equation: $2x(1-x)\frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} + 3y = 0$ 8

(b) Find regular singular points of the differential equation:

$x(x-2)^2 y'' + 2(x-2)y' + (x+3)y = 0$ 6

4. (a) Prove that $P_n(x)$ is the coefficient of z^n in the expansion of

$(1-2xz+z^2)^{-\frac{1}{2}}$ in ascending powers of x. 8

(b) Express $f(x)=4x^3+6x^2+7x+2$ in terms of Legendre polynomials. 6

UNIT–III

5. (a) Show that Bessel's function $J_n(x)$ is an even function when n is even and is odd function when n is odd. 8

(b) Prove that: 3+3=6

(i) $J_{1/2}(x)=\sqrt{\dfrac{2}{\pi x}}\sin x$

(ii) $J_{-1/2}(x)=\sqrt{\dfrac{2}{\pi x}}\cos x$

6. (a) Show that: $\int_0^\infty f_m(x)f_n(x)dx=\int_0^\infty e^{-x}\dfrac{L_m(x)}{m!}\dfrac{L_n(x)}{n!}dx=\delta_{m,n}$ over the interval $0\le x\le\infty$, when $\delta_{m,n}=\begin{cases}0\ for\ m\neq n\\1\ for\ m=n\end{cases}$.

(Here, $L_n(x)$ represents Laguerre polynomial of degree n) 8

(b) Prove that: $H_n(-x)=(-1)^n H_n(x)$, where $H_n(x)$ is the Hermite polynomial of degree n. 6

UNIT–IV

7. (a) Evaluate using beta and gamma functions: $\int_0^1\left(\dfrac{x^3}{1-x^3}\right)^{1/2}dx$ 6

(b) Prove that: 4+4=8

(i) $\Gamma(n+1) = n\Gamma(n)$

(ii) $\Gamma(n+1) = n!$

8. (a) Derive the normal law of errors and calculate the probable error of an observation. 8

(b) Show that: $\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(a+x)^{m+n}} dx = \frac{\beta(m,n)}{a^n (1+a)^m}$ 6

UNIT–V

9. (a) Solve the following equation by the method of separation of variables: $\frac{\partial^2 u}{\partial x \partial t} = e^{-t} \cos x$ 8

(b) A tight string of length π, fixed at both ends, is given an initial displacement $f(x)$ and an initial velocity $g(x)$. Find the expression $y(x,t)$ at the time *t*. 6

10. (a) Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ in a rectangle in the xy-plane with $u(x,0) = 0,\ u(x,b) = 0,\ u(0,y)$ and $u(a,y) = f(y)$ parallel to y-axis. 10

(b) Derive the one-dimensional diffusion equation. 4