2022

B.A./B.Sc.

First Semester

CORE – 1

PHYSICS

Course Code: PHC 1.11

(Mathematical Physics)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five *questions, taking* one *from each unit.*

UNIT–I

1. (a) If $f(z)$ is an analytic function in the powers of $(\xi - a)^n$, then write the Taylor's series expansion for $f(\xi)$. 2

   (b) Obtain the solution for the second order homogeneous equation with constant coefficients. 7

   (c) Show that the given differential equation is an exact differential equation and solve it: $\left(1+e^{x/y}\right)+e^{x/y}\left(1-\frac{x}{y}\right)\frac{dx}{dy}=0$. 5

2. (a) If $y_1 = e^{-x}\cos x,\ y_2 = e^{-x}\sin x$ and $\frac{d^2y}{dx^2}+2\frac{dy}{dt}+2y=0$, then
   (i) calculate the Wronskian determinant
   (ii) apply Wronskian test to check that y_1 and y_2 are linearly independent. 5

   (b) Find the point upon the plane $ax+by+cz=p$ at which the function $f=x^2+y^2+z^2$ has a minimum value and find this minimum f. 6

   (c) Solve the equation: $\frac{d^2y}{dx^2}+6\frac{dy}{dx}+9y=5e^{3x}$ 3

UNIT–II

3. (a) If $\phi = 3x^2y - y^3z^2$, find grad ϕ at the point (1,–2,–1). 3

(b) Show that when three vectors lie in a plane, the volume of the parallelopiped is zero. 7

(c) Find the directional derivative of $\phi = x^2 y + xz$ at (1,2,–1) in the direction of $\vec{A} = 2\hat{i} - 2\hat{j} + \hat{k}$. 4

4. (a) If $\vec{A}$ is a constant vector and $\vec{R} = x\hat{i} + y\hat{j} + z\hat{k}$, then prove that $curl\left[\left(\vec{A}.\vec{R}\right)\vec{A}\right] = \vec{A} \times \vec{R}$ 4

(b) Find the value of the constant 'a' so that the vector, $\vec{A} = (x+3y)\hat{i} + (2y+3z)\hat{j} + (x+az)\hat{k}$ is a solenoidal vector. 4

(c) Derive the expression for the divergence of a vector field. 6

UNIT–III

5. (a) If $\vec{V} = (3x^2 + 6y)\hat{i} - 14yz\hat{j} + 20xz^2\hat{k}$, then evaluate $\int_C \vec{V}.d\vec{r}$, where C is a straight line joining (0,0,0) and (1,1,1). Given $x = t, y = t^2, z = t^3$. 7

(b) State and prove Gauss's divergence theorem. 7

6. (a) Evaluate $\iiint \left(\vec{\nabla} \times \vec{F}\right) dV$, where V is the closed region bounded by the planes $x = 0, y = 0, z = 0$, $2x + 2y + z = 4$ and $\vec{F} = (2x^2 - 3z)\hat{i} - 2xy\hat{j} - 4x\hat{k}$. 7

(b) Use the divergence theorem to show that,

$$\iint \nabla(x^2 + y^2 + z^2) d\vec{S} = 6V$$ 7

UNIT–IV

7. (a) Derive the expression of $\nabla\phi$ in the orthogonal curvilinear coordinates. 8

(b) Express the following in spherical coordinates:

(i) $\vec{\nabla} \times \vec{A}$

(ii) $\vec{\nabla}^2\psi$ 3+3=6

8. (a) Show that in the orthogonal coordinates: 3×2=6

(i) $\nabla.(A_1 e_1) = \frac{1}{h_1 h_2 h_3} \frac{\partial}{\partial u_1}(A_1 h_2 h_3)$

(ii) $\nabla \times (A_1 e_1) = \frac{e_2}{h_3 h_1} \frac{\partial}{\partial u_3}(A_1 h_1) - \frac{e_3}{h_1 h_2} \frac{\partial}{\partial u_2}(A_1 h_1)$

(b) Express $div\vec{V}$ in the cylindrical coordinates. 8

UNIT–V

9. (a) Obtain the expression for the binomial distribution by the example of repeated tosses of a dice. 8

(b) State Bayes' theorem. 2

(c) Using the definition of a continuous probability distribution for the variance σ^2 of a continuous random variable, show that:

$\sigma^2 = \langle X^2 \rangle - \langle X \rangle^2$ 4

10. (a) Define Dirac delta function. 2

(b) Show that:

$$\int_{-\infty}^{+\infty} f(t)\delta'(t-a)dt = -f'(a)$$ 4

(c) Evaluate the following:

(i) $\int_{0}^{\infty} e^{-3t}\delta(t-4)dt$ 2

(ii) $\int_{-\infty}^{+\infty} e^{-3t}\delta(t-2)dt$ 2

(d) Show that in the limit $\sigma \to 0$, the rectangular function $R_\sigma(x)$ has all the properties of the Dirac delta function. 4