2022

M.Sc.

Third Semester

DISCIPLINE SPECIFIC ELECTIVE – 01

MATHEMATICS

Course Code: MMAD 3.11

(Classical Mechanics)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) What type of difficulties arises due to the constraints in the solution of mechanical problems and how are these removed? 1+1=2

   (b) Show that the Lagrange's equation $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_k}\right)-\frac{\partial T}{\partial q_k}=G_k$ can be written as $\frac{\partial \dot{T}}{\partial \dot{q}_k}-2\frac{\partial T}{\partial q_k}=G_k$ 5

   (c) Two wheels of radius a are mounted on the ends of a common axle of length b such that the wheels rotate independently. The whole combination is rolling without slipping on a plane. Show that there are two non-holonomic equations of constraints

   $$\cos\theta dx+\sin\theta dy=0, \sin\theta dx-\cos\theta dy=\frac{1}{2}a\left(d\phi+d\phi'\right)$$

   and one holonomic constraints equation $\theta=C-\frac{a}{b}\left(\phi-\phi'\right)$

   where θ, ϕ, ϕ' have meaning similar to those in the problem of a single vertical disk and C is a constant 3½+3½=7

2. (a) Prove that in a simple conservative dynamical system, sum of kinetic energy and potential energy is conserved. 7

   (b) Discuss the motion of a particle of mass m moving in a polar plane by using Lagrange's equation. 7

UNIT–II

3. (a) Prove that in absence of a given component of applied force, the corresponding component of linear momentum is constant. 7

   (b) Derive Hamilton's equations of motion in generalised coordinates. 7

4. (a) Derive Hamilton's equation of motion in polar coordinates. 7

   (b) Derive Hamilton's equation of motion for a charged particle in an electromagnetic field 7

UNIT–III

5. (a) Derive Hamilton's principle for non-conservative system from D'Alembert's principle and hence deduce from it the Hamilton's principle for conservative system. 5+2=7

   (b) Show that the shortest distance between two points in a plane is a straight line. 7

6. (a) Deduce Lagrange's equation from variational principle for non-conservative system with holonomic constraints. 7

   (b) State and verify the principle of least action. 1+6=7

UNIT–IV

7. (a) Discuss one-dimensional harmonic oscillator as an example of canonical transformations. 7

   (b) Discuss the integral invariant of Poincaré. 7

8. (a) The transformation equations between two sets of coordinates are represented by $P = 2\left(1+\sqrt{q}\cos p\right)\sqrt{q}\sin p$

   and $Q = \ln\left(1+\sqrt{q}\cos p\right)$

   (i) Show that the transformation is canonical. 4

   (ii) Show that the generating function of this transformation is

   $F_3 = -\left(e^Q - 1\right)^2 \tan p$ 3

   (b) Show that Poisson bracket is invariant under canonical transformation. 7

UNIT–V

9. (a) Define Euler's angles and obtain an expression for complete transformation matrix. 7

   (b) Derive Euler's equation of motion for a rotating rigid body with a fixed point. 7

10. (a) Show that angular momentum $\vec{J}$ of a rotating rigid body can be expressed as $\vec{J} = I\vec{\omega}$ where I is the inertia tensor and $\vec{\omega}$ is the angular velocity. 7

    (b) Consider a rectangular parallelopiped of uniform density ρ, mass M with sides a,b,c. For origin O at one corner, find the moments and products of inertia of the parallelopiped by taking the coordinate axes along the edges. If $a = b = c$, determine the inertia tensor.

3+2+2=7
