2022

M.Sc.

Third Semester

CORE – 09

MATHEMATICS

Course Code: MMAC 3.11

(Numerical Analysis)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five *questions, taking* one *from each unit.*

UNIT–I

1. (a) Derive the Regula-Falsi iteration formula. 4
   (b) Discuss the rate of convergence of Newton-Raphson method. 5
   (c) Determine a real root of $\tan x = x$ using secant method by taking appropriate initial value. 5

2. (a) Define relative error? If $f(x, y, z) = 3\dfrac{xy}{z^3}$ and errors in x, y, z be 0.001. Compute the maximum relative error in $f(x, y, z)$ when $x = y = z = 1$. 3
   (b) Perform three iterations of the Chebyshev's method to find the approximate value of $\frac{1}{7}$. (take $x_o = 0.1$) 5
   (c) Find the root of the equation $f(x) \equiv x^4 - x - 10 = 0$ using multipoint iteration method. (perform four iterations) 6

UNIT–II

3. (a) Find the inverse of the matrix using Gauss-Jordan method. 7

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

(b) What is Doolittle's & Crout's method? Solve the system of equation by triangularization method. 1+6=7

$$4x + y + z = 4$$
$$x + 4y - 2z = 4$$
$$3x + 2y - 4z = 6$$

4. (a) Find the inverse of the matrix by partition method. 7

$$\begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$$

(b) Find the largest eigenvalue and its corresponding eigenvector of the matrix correct to 3 decimal places using power method. 7

$$\begin{bmatrix} 4 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 1 & 4 \end{bmatrix}$$

UNIT–III

5. (a) Construct the Hermite interpolation that fits the data and interpolate at $x = 0.5$ and $x = 1.5$. 8

x	$f(x)$	$f'(x)$
0	4	–5
1	–6	–14
2	–22	–17

(b) Obtain the piecewise cubic interpolating polynomials for the function define by the given data. 6

x	–5	–4	–2	0	1	3	4
$f(x)$	215	–94	–334	–350	–344	–269	–94

Hence interpolate at $x = -3.0$ and $x = 2.0$.

6. (a) For the data $f(10) = 1.1585, f(20) = 1.2817, f(30) = 1.3660$.

Construct the Lagrange interpolating and calculate $f(15)$. 6

(b) Use Sterling's formula to find $y(35)$, given that
$y(20)=215, y(30)=439, y(40)=346, y(50)=247$. 6

(c) Prove the following relation: 1+1=2

(i) $\delta = \Delta E^{-1/2} = \nabla E^{1/2}$ (ii) $\mu^2 = 1 + \frac{1}{4}\delta^2$

UNIT–IV

7. (a) Find the value of $f'(9)$ and $f''(9)$ for the given data 4
$f(6)=1.556, f(7)=1.690, f(9)=1.908, f(12)=2.158$.

(b) The function $y = \sin x$ is tabulated below: 4

x	0.7	0.8	0.9	1.0	1.1	1.2	1.3
y	0.644218	0.717356	0.783327	0.841471	0.891207	0.932039	0.963558

Find the derivative at the point $x = 1.3$ and compute its actual error.

(c) Find $f''(5)$ using divided difference formula for the data: 6
$f(3)=-13, f(5)=23, f(11)=899, f(27)=17315,$
$f(34)=35606$

8. (a) Using Simpson's 1/3 rule evaluate $I = \int_0^1 \frac{dx}{x^2+6x+10}$ with 4 & 8 subintervals. 7

(b) Evaluate $\int_0^2 \frac{x^2+2x+1}{1+(x+1)^4}dx$ by Gauss-Legendre three point rule. 7

UNIT–V

9. (a) Consider the initial value problem $\frac{dy}{dx} = x(y+1), y(0)=0$,

Compute $y(0.5)$ with $h = 0.1$ using Euler method. If the exact solution is $y = -1 + 2e^{x^2/2}$. Find the magnitude of actual errors. 7

(b) Solve the initial value problem $\frac{dy}{dx} = 2x + 3y, y(0) = 1$, using Taylor series method with $h = 0.2$ over the interval $[0,1]$. Compare with the exact solution. 7

10. (a) Reduce the second order inital value problem $y'' + 3y' + 2y = e^{2t}$, with $y(0) = 1, y'(0) = 1$ to a system of first order inital value problems and find the value of $y(1), y'(1)$ (taking $h = 0.5$) using Runge-Kutta method of fourth order. 7

(b) Determine the value of $y(0.4)$ by Milne-Simpson method given that $y' = xy + y^2, y(0) = 1$ with $h = 0.1$. 7
