2022
B.A./B.Sc.
Third Semester
SKILL ENHANCEMENT COURSE – 1
MATHEMATICS
Course Code: MAS 3.11
(Logic & Sets)

Total Mark: 35   *Pass Mark: 14*
Time: 2 hours

Answer <u>five</u> *questions, taking* <u>one</u> *from each unit.*

UNIT–I

1. (a) Define conjunction of two propositions p and q. Also construct the corresponding truth table.   2
    (b) Determine the truth values of each of the following statements: 2
        (i)   If $1 + 1 = 3$, then $2 + 2 = 4$
        (ii) $4 + 1 = 2$ or $8 \div 2 = 4$
    (c) State the converse, contrapositive and inverse of the conditional statement "I come to class whenever there is going to be a quiz".   3

2. (a) Construct the truth table of the compound proposition
        $$(p \wedge \neg q) \rightarrow (p \wedge q)$$ 3
    (b) Given that p and q are true and r and s are false, find the truth values of the following propositions: 2×2=4
        (i) $(p \wedge s) \vee (q \vee r)$
        (ii) $(\neg(p \wedge q) \vee \neg s) \vee (((\neg p \wedge q) \vee \neg r) \vee s)$

UNIT–II

3. (a) Prove that the following propositions are tautologies: 1×2=2
        (i) $\neg(p \wedge s) \vee q$
        (ii) $p \rightarrow (p \vee q)$

(b) Let the set of integers be the universe of discourse. Find the truth values of the statements "for every $x \in \mathbb{Z}, x^2 = 1$" and "$(\exists x \in \mathbb{Z}) x^2 = x$". 2

(c) Let $P(x) : x$ can speak Tamil
$\quad\quad Q(x) : x$ knows French
Translate each of the following statements into symbols using quantifiers, variables, and predicate symbols. $1 \times 3 = 3$
(i) There is a student who can speak Tamil and who knows French.
(ii) Every student either speaks Tamil or French.
(iii) No student can speak Tamil or knows French.

4. (a) Write the symbolic form and the negation of the following statements:
(i) All integers are greater than 8 $2 \times 2 = 4$

(ii) For all real numbers x, if $x > 3$ then $x^2 > 9$.

(b) Using truth table, prove that $p \vee (q \wedge r)$ and $(p \vee q) \wedge (q \vee r)$ are logically equivalent. 3

UNIT–III

5. (a) If $A = \{a, b, c\}$, $B = \{d, x, y\}$ and $C = \{x, y\}$, find the number of proper subsets for the set $(A \cap B) \cup C$ and the set $A \cap (B \cup C)$? 3

(b) If $A \cup B = A \cup C$ and $A \cap B = A \cap C$. Prove that $B = C$. 4

6. (a) Prove that the set $\{1, 4, 9, 16, 25, ...\}$ is countable. 3
(b) A computer company receives 350 applications from computer graduates for a job. Suppose that 220 of these applicants majored in computer science, 147 majored in business, and 51 majored both in computer science and in business. How many of these applicants majored neither in computer science nor in business. 4

UNIT–IV

7. (a) Show that the sets $A - B$ and $A \cap B$ are disjoint. 3
(b) If $A, B \subseteq U$, then A and B are disjoint if and only if $A \cup B = A \Delta B$. 4

8. (a) Define power set of a set. Find the number of elements in the power set of a set $A = \{x \in \mathbb{R} \mid x^2 + 5x + 6 = 0\}$.   1+3=4

   (b) Define symmetric difference of two sets. Prove that
$A \cup (A \cap B) = A$.   1+2=3

UNIT–V

9. (a) Find the number of relations on a set with n elements.   2
   (b) Prove that an equivalence relation R in a non-empty set A partitions A and conversely, a partition of A defines an equivalence relation in A.
    5

10. (a) Define symmetric relations with proper example. Give an example of a relation on a set that is neither symmetric nor antisymmetric.   3
   (b) Show that the relation $R = \{(a,b \mid a \equiv b(\mathrm{mod}\, m)\}$ is an equivalence relation on the set of integers.   4

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