**2022**

**B.A./B.Sc.**

**Third Semester**

GENERIC ELECTIVE – 3

**MATHEMATICS**

*Course Code: MAG 3.11*

(Vectors & Analytical Geometry)

*Total Mark: 70* *Pass Mark: 28*

*Time: 3 hours*

*Answer five questions, taking one from each unit.*

## UNIT–I

1. (a) Find a vector equation of the line tangent to the curve: 3+3=6

   (i) $\bar{r}(t) = (2t-1)\hat{i} + \sqrt{3t+1}\,\hat{j}$ at $P_o(-1,-2)$

   (ii) $\bar{r}(t) = 4\cos t\,\hat{i} - 3t\,\hat{j} + 2\sin t\,\hat{k}$ at $P_o\left(2,-\pi,\sqrt{3}\right)$

   (b) Use the formula $\bar{B}(t) = \bar{T}(t)\times\bar{N}(t)$ to find $\bar{B}(t)$ and check you answer by using your formula for $\bar{B}(t)$ which is expressed in terms of $\bar{r}(t)$ where $\bar{r}(t) = 3\sin t\hat{i} + 3\cos t\hat{j} + 4t\hat{k}$. 4

   (c) Find the curvature and radius of curvature at $t = \dfrac{\pi}{2}$ for $\bar{r}(t) = 3\cos t\,\hat{i} + 4\sin t\,\hat{j} + t\,\hat{k}$. 4

2. (a) Solve the vector initial value problem for $y(t)$ by integrating and using the initial conditions: 2+3=5

   (i) $y'(t) = 2t\,\hat{i} + 3t^2\,\hat{j},\ y(0) = \hat{i} - \hat{j}$

   (ii) $y''(t) = \hat{i} + e^t\,\hat{j},\ y(0) = 2\hat{i},\ y'(0) = \hat{j}$

   (b) Find $\kappa(t)$ for the circular helix $x = a\cos t, y = a\sin t, z = ct, a > 0$. 4

   (c) If $\bar{f} = x^2y\hat{i} - 2xz\hat{j} + 2yz\hat{k}$, find 1+2+2=5

   (i) div $\bar{f}$ (ii) curl $\bar{f}$

   (iii) curl curl $\bar{f}$

## UNIT–II

3. (a) Evaluate the line integral w.r.t. $s$ along the curve $C$: $\int_C \frac{e^{-z}}{x^2+y^2}\,ds$,

$C$: $\bar{r}(t) = 2\cos t\,\hat{i} + 2\sin t\,\hat{j} + t\,\hat{k},\ (0 \le t \le 2\pi)$. 3

(b) Evaluate the line integral along the curve C:

$\int_c (x+2y)\,dx + (x-y)\,dy, C: x = 2\cos t, y = 4\sin t\left(0 \le x \le \frac{\pi}{4}\right)$ 3

(c) Evaluate $\int_c \bar{F}.d\bar{r}$ along the curve $C$, where

$\bar{F}(x,y) = (x^2+y^2)^{-3/2}(x\hat{i} + y\hat{j})$,

$C: \bar{r}(t) = e^t \sin t\,\hat{i} + e^t \cos t\,\hat{j}\ \ (0 \le t \le 1)$ 3

(d) Give the statement of the theorem related to the conservative field test and use it to check whether $\bar{F}$ is a conservative vector field. If so, find the potential function for

$\bar{F}(x,y) = (\cos y + y\cos x)\hat{i} + (\sin x - x\sin y)\hat{j}$ 5

4. (a) State the fundamental theorem of line integral and use it to solve by showing that the integral is independent of the

path. $\int_{(2,-2)}^{(-1,0)} \left(2xy^3\,dx + 3y^2x^2\,dy\right)$ 4

(b) State and prove the Green's theorem and use it to

solve $\oint_C (x\cos y\,dx - y\sin x\,dy)$, where $C$ is the square with

vertices $(0,0), \left(\frac{\pi}{2}, 0\right), \left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\left(0, \frac{\pi}{2}\right)$. 10

## UNIT–III

5. (a) Evaluate $\iint_S \bar{F}, \hat{n}d\bar{S}$, where $\bar{F} = yz\hat{i} + zx\hat{j} + xy\hat{k}$ and $S$ is that part of the surface of the sphere $x^2 + y^2 + z^2 = 1$ which lies in the first octant. 5

(b) Evaluate $\iiint_V \varphi d\bar{V}$, where $\varphi = 45x^2 y$ and $V$ is the closed region bounded by $4x + 2y + z = 8, x = y = z = 0$. 4

(c) Verify Stokes' theorem for $\bar{F} = y\hat{i} + z\hat{j} + x\hat{k}$ where $S$ is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and $C$ is its boundary. 5

6. (a) Prove that $\iint_S (\nabla\varphi) \times \hat{n} d\bar{S} = 0$ for a closed surface $S$. 4

(b) If $\bar{F} = x\hat{i} - y\hat{j} + (z^2 - 1)\hat{k}$, find the value of $\iint_S \bar{F}.\hat{n} d\bar{S}$ where $S$ is the closed surface bounded by the plane $z = 0, z = 1$ and the cylinder $x^2 + y^2 = 4$. 4

(c) Verify Stokes' theorem for $\bar{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ taken round the rectangle bounded by $x = \pm a, y = 0, y = b$ 6

## UNIT–IV

7. (a) Find the coordinate of a point in a plane when the origin is shifted to a new point $(h, k)$, the new axis remaining parallel to the original axis. 5

(b) By transforming to parallel axis through a properly chosen point, prove that the equation $12x^2 - 10xy + 2y^2 + 11x - 5y + 2 = 0$ reduced to one containing only terms of the second degree. 5

(c) Prove that the normal at an end of a latus rectum passes through an end of the minor axis, if $e^4 + e^2 = 1$. 4

8. (a) Find the lengths of the semi-axis of the conic

$$7x^2 + 52xy - 32y^2 = 180.$$ 4

(b) A point is such that the perpendicular from the centre on its polar with respect to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is constant and equal to $C$; show that its locus is the ellipse $\frac{x^2}{a^4} + \frac{y^2}{b^4} = \frac{1}{c^2}$. 4

(c) Prove that the locus of the middle points of chords of contact of perpendicular tangents to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is given by

$$\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)^2=\frac{x^2+y^2}{a^2+b^2}.$$ 6

## UNIT–V

9. (a) Find the coordinates of a point which divides the join of the points $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$ in the ratio $m{:}n$. 4

(b) Find the angle between two diagonals of a cube. 5

(c) Prove that the four points (1, 3, –1), (3, 5, 1), (0, 2, –2) and (2, 1, –2) are coplanar and find the equation of the plane. 5

10. (a) Find the equation of the plane which passes through the intersection of the planes $x-2y-3z=4$, $2x+3y-z=1$ and is perpendicular to the plane $3x-y+2z+5=0$. 4

(b) Prove that the lines $\frac{x+3}{5}=\frac{y-1}{2}=\frac{z-1}{3}$ and $\frac{x-3}{1}=\frac{y-7}{4}=\frac{z-6}{2}$ are coplanar and find their point of intersection and the equation of the plane in which they lie. 5

(c) Find the shortest distance between the lines

$3x-9y+5z=0=x+y-z$ and

$6x+8y+3z-13=0=x+2y+z-3$ 5