2022

B.A./B.Sc.

First Semester

GENERIC ELECTIVE – 1

MATHEMATICS

Course Code: MAG 1.11

(Calculus)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) If $y=\dfrac{x}{x^2+a^2}$, find y_n by partial fraction method. 5

   (b) If $y=e^{ax}\sin(bx+c)$, show that

   $$y_n=\left(a^2+b^2\right)^{\frac{n}{2}}e^{ax}\sin\left(bx+c+n\tan^{-1}\frac{b}{a}\right).$$ 5

   (c) Separate the intervals in which the polynomial $f(x)=x^3+8x^2+5x-2$ is increasing or decreasing using first derivative test. 4

2. (a) If $y=x\log\left(\dfrac{x-1}{x+1}\right)$, prove that

   $$y_n=(-1)^n(n-2)!\left[\frac{(x-n)}{(x-1)^n}-\frac{(x+n)}{(x+1)^n}\right].$$ 5

   (b) If $y=a\cos(\log x)+b\sin(\log x), x>0$, show that $x^2y_{n+2}+(2n+1)xy_{n+1}+\left(n^2+1\right)y_n=0$ using Leibnitz's theorem. 5

   (c) Given the position $s(t)=2t^2-5t+2, 0\le t\le 3$ of a body moving on a coordinate line with *s* in meters and *t* in seconds, find

   (i) displacement and average velocity 2

   (ii) speed and acceleration at the endpoints of the interval 2

UNIT–II

3. (a) State and prove Lagrange's mean value theorem. 1+4=5

(b) Find the asymptotes of $xy^2 - y^2 - x^3 = 0$. 5

(c) Evaluate $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{1}{x}}$. 4

4. (a) By using mean value theorem, show that

$\frac{v-u}{1+v^2} < \tan^{-1} v - \tan^{-1} u < \frac{v-u}{1+u^2}$, $0 < u < v$ and deduce

$\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$. 5

(b) Verify Rolle's theorem for the function $f(x) = x(x+3)e^{-\frac{x}{2}}$ in $[-3,0]$. 5

(c) Use mean value theorem in making numerical approximation of $(28)^{\frac{1}{3}}$. 4

UNIT–III

5. (a) State and prove Maclaurin's theorem with Lagrange's form of remainder. 5

(b) Expand $\sin x$ in a finite series in powers of x with remainder in Lagrange's form. 5

(c) Find $\lim_{x\to 0}\left(\frac{1}{\sin x} - \frac{1}{x}\right)$ using Taylor series. 4

6. (a) Use Maclaurin's theorem to find the expansion of $\log\left(1+e^x\right)$ in ascending powers of x to the terms containing x^4. 5

(b) Show that $\lim_{h\to 0}\theta = \frac{1}{n+1}$, where θ is given by

$$f(a+h) = f(a) + hf'(a) + \ldots + \frac{h^{n-1}}{(n-1)!}f^{n-1}(a) + \frac{h^n}{n!}f^n(a+\theta h),$$

provided that $f^{n+1}(x)$ is continuous at a, $f^{n+1}(a) \neq 0$. 5

(c) Evaluate $\lim_{x\to 0} \frac{\sin x - \tan x}{x^3}$ using Taylor series. 4

UNIT–IV

7. (a) If $I_n = \int_0^{\pi/2} \sin^n x\, dx$, where n is a positive integer greater than 1,

show that $I_n = \frac{n-1}{n} I_{n-2}$ and evaluate $\int_0^{\pi/2} \sin^5 x\, dx$. 5

(b) If m and n are positive integers, show that

$$\int_0^{\pi} \cos mx \sin nx\, dx = \begin{cases} \frac{2n}{n^2 - m^2}, & \text{when } (n-m) \text{ is odd} \\ 0, & \text{when } (n-m) \text{ is even} \end{cases}$$ 5

(c) Show that $\int_0^{\pi} \frac{dx}{3 + 2\sin x + \cos x} = \frac{\pi}{4}$. 4

8. (a) If m, n are positive integers, then prove that

$$\int_0^1 x^{m-1} (1-x)^{n-1} dx = \frac{1.2.3\ldots(m-1)}{n(n+1)\ldots(n+m-1)}.$$ 5

(b) Show $\int_0^{\infty} e^{-ax} x^n dx = \frac{n!}{a^{n+1}}$ where a is a positive quantity and n is a positive integer. 5

(c) Evaluate $\int \sqrt{\frac{a+x}{a-x}} dx$. 4

UNIT–V

9. (a) Use cylindrical shell to find the volume of the solid region generated when the region R in the first quadrant enclosed between $y = x$ and $y = x^2$ is revolved about the y-axis. 5

(b) Find the area of the surface generated when the arc $y = x^2$ between $x = 0$ and $x = \sqrt{2}$ is revolved about the y-axis. 4

(c) Show that the volume of the solid formed by the rotation of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the major axis is $\frac{4}{3}\pi ab^2$ and about the minor axis is $\frac{4}{3}\pi a^2 b$. 5

10. (a) Show that the circumference of a circle with radius r is $2\pi r$. 5

(b) Find the volume generated by revolving the cardioid $r = a(1 - \cos\theta)$ about the initial line. 5

(c) Find the surface of the sphere generated by the circle $x^2 + y^2 = a^2$ about the x-axis. 4