2022

B.A./B.Sc.

Fifth Semester

DISCIPLINE SPECIFIC ELECTIVE – 2

MATHEMATICS

Course Code: MAD 5.21

(Boolean Algebra & Automata Theory)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five *questions, taking* one *from each unit.*

UNIT–I

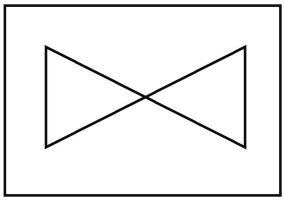
1. (a) Define poset with example. 2

   (b) Draw all possible ordered sets with five elements. 5

   (c) Draw the diagram of $M_3 \oplus M_4, 2^4, 1 \oplus (1 \cup 1)$ 1+1+1=3

   (d) Prove that two finite ordered set P & Q are order-isomorphic if and only if they can be drawn with identical diagram. 4

2. (a) Draw and label a diagram of the ordered sets of down sets for the ordered set P given by the diagram. 6



   (b) Define order-preserving with example. Let $\varphi : P \to Q$ and $\psi : Q \to R$ be order -preserving maps. Then show that the composite map $(\psi \circ \varphi)$ given by $(\psi \circ \varphi)(x) = \psi(\varphi(x))$ for $x \in P$ is also order-preserving. 1+2=3

(c) Let $(L, \wedge, \vee)$ be a non-empty set equipped with two binary operations which satisfy the axioms of join and meet. Then, prove that $\forall a, b, c, d \in L$ 2½×2=5

(i) $a \leq b \Rightarrow a \vee c \leq b \vee c$ and $a \wedge c \leq b \wedge c$

(ii) $a \leq b$ and $c \leq d \Rightarrow a \vee c \leq b \vee d$ and $a \wedge c \leq b \wedge d$

UNIT–II

3. (a) State and prove De Morgan's law in Boolean algebra. 1+2=3
   (b) Define sublattice. Give an example to show that the subset of a lattice L is a lattice on its own but not a sublattice of L. 1+2=3
   (c) Define complemented lattice. Prove that in a distributive lattice, each element has at most one complement. 1+3=4
   (d) Prove that direct product of Boolean algebra is a Boolean algebra. 4

4. (a) Define disjunctive and conjunctive normal form with examples. 1+1=2
   (b) Minimize the polynomial $w'x'y'z' + w'xy'z + wx'y'z' + wx'y'z + wx'yz' + wx'yz + wx'yz + wxyz' + wxyz$ using Quine-McCluskey method. 6
   (c) A committee of three judges deciding on the acceptance or rejection of a competitor in a game are provided with buzzers in which the push is to indicate acceptance. Design a circuit so that a bell will ring when there is a majority of vote for acceptance. 6

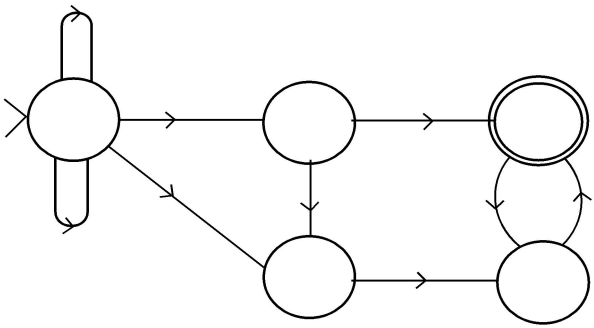
UNIT–III

5. (a) Define Kleene star of a language. 2
   (b) Find the regular expression and draw its finite automata for the given formal language. 2×2=4

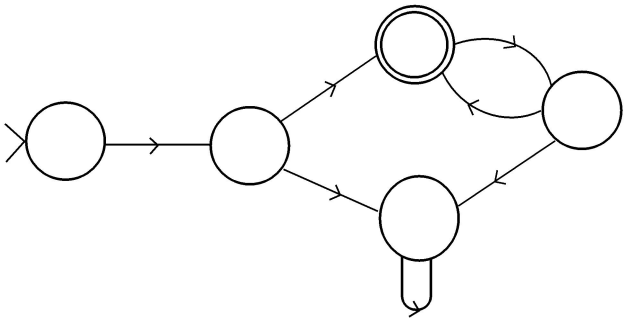
   (i) $L = \{w \in \{0,1\}^* : w \text{ has even number of zeros}\}$

   (ii) $L = \{w \in \{0,1\}^* : |w| \text{ is divisible by } 4\}$

(c) Construct an equivalent deterministic finite automata (DFA) for the given non-deterministic finite automata (NFA). 8



6. (a) Find the regular expression for the language accepted by the deterministic finite automaton whose state diagram is 7



(b) Show that union of two regular language is regular. 2

(c) State the pumping theorem for regular language. Show that

$L = \{ww^R : w \in \{a,b\}^*\}$ is not a regular language. 1+4=5

UNIT–IV

7. (a) Define linear and nonlinear grammar. 1+1=2

(b) Construct a PDA that accepts the language 4

$L = \{w \in \{a,b\}^* : w$ has the same number of a's and b's$\}$

(c) Prove that every regular language is a context free language. 4

(d) Construct a PDA for the given context free grammar
$G = (V, \Sigma, R, S)$ where $V = \{S,a,b,c\}$, $\Sigma = \{a,b,c\}$,
$R = \{S \to aSa, S \to bSb, S \to c\}$ 4

8. (a) Define ambiguous grammar with example. 2
(b) Let G be a grammar with the rules R given by 6
$S \to aB/bA,\ A \to a/aS/bAA, B \to b/bS/aBB$
Then, find the leftmost and rightmost derivation for the string *aaabbabbba*. Also, draw the parse tree.
(c) Show that $L = \{ww : w \in \{0,1\}^*\}$ is not a context free language. 6

UNIT–V

9. (a) Discuss the uses of Turing machine by giving an example along with a sample computation. 4
(b) Construct and draw a Turing machine that accepts the language
$L = \{a^n b^n c^n : n \geq 0\}$ 5
(c) Construct a Turing machine which compute the multiplication function and trace the input $\#I^3 \#I^2 \#$ 4

10. (a) Define the following with example: 3×3=9
(i) Decidable
(ii) Undecidable
(iii) Post correspondence problem
(b) Draw the left shifting standard machine. 2
(c) What do you mean by halting of Turing machine? 3