2022

B.A./B.Sc.

Fifth Semester

CORE – 11

MATHEMATICS

Course Code: MAC 5.11

(Multivariate Calculus)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five *questions, taking* one *from each unit.*

UNIT–I

1. (a) Show that the function 5

$$f(x,y)=\begin{cases}\dfrac{xy}{\sqrt{x^2+y^2}}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$$

is continuous at the origin.

(b) If $f(x,y)=x^2y+e^xy^2$, find f_x and f_y. 4

(c) Show that the function 5

$$f(x,y)=\begin{cases}xy\dfrac{x^2-y^2}{x^2+y^2}, & \text{if } x^2+y^2\neq 0\\ 0, & x=y=0\end{cases}$$

is differentiable at the origin.

2. (a) By using the definition of limit, show that 5

$$\lim_{(x,y)\to(0,0)}=\frac{x^2+y^2}{xy}$$ does not exist.

(b) By definition of partial derivatives, evaluate $f_x(2,0)$ and $f_y(1,-1)$ for the function $f(x,y)=5+2xy-3y^2+x^2y$. 5

(c) By using chain rule, find $\frac{dw}{dt}$ for $w = xe^y + y\sin x; x = t, y = t^2$. 4

UNIT–II

3. (a) Find the points on the curve $x^2 y = 2$ nearest to the origin. 4

(b) Find curl $\vec{F}$ and div $\vec{F}$ for $\vec{F}(x, y, z) = 2x^2 z\hat{i} + 2xy^3\hat{j} + 3yz^2\hat{k}$. 5

(c) Let $\phi = x^2 yz - 4xyz^2$. Find the directional derivatives of ϕ at P(1,3,1) in the direction of $2\hat{i} - \hat{j} - 2\hat{k}$. 5

4. (a) Find the gradient of $\phi(x, y, z) = 3x^2 y - y^2 z^2$ at the point (1,–2,–1). 4

(b) Find the tangent plane and normal line to the surface $x^2 + xyz - z^3 = 1$ at the point P(1,1,1). 5

(c) Find the direction in which the function $f(x, y) = x^2 + \cos xy$ increases or decreases most rapidly at $P(1, \frac{\pi}{2})$. 5

UNIT–III

5. (a) Evaluate $\int_{-1}^{0}\int_{-1}^{1}(x + y + 1)\,dxdy$ 4

(b) Evaluate $\int_{1}^{2}\int_{y}^{y^2} dxdy$ 5

(c) Evaluate $\int_{0}^{1}\int_{0}^{1-z}\int_{0}^{2} dxdydz$ 5

6. (a) Evaluate $\int_{0}^{2}\int_{0}^{x} y\,dxdy$ by changing into polar form. 5

(b) Change the order of integration $\int_0^1 \int_1^{e^x} dxdy$. 4

(c) Evaluate $\int_0^{\pi} \int_0^{\pi} \int_r^{2\sin\phi} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$ 5

UNIT–IV

7. (a) Let $A = (3x^2 + 6y)\hat{i} - 14yz\hat{j} + 20xz^3\hat{k}$. Evaluate $\int_C A dr$ from (0,0,0) to (1,1,1) along the curve $C: x = t, y = t^2, z = t^3$. 5

(b) Find the total work done in moving a particle in the force given by $\vec{F} = z\hat{i} + z\hat{j} + x\hat{k}$ along the helix C given $x = \cos t, y = \sin t, z = t$ by from $t = 0$ to $t = \frac{\pi}{2}$. 5

(c) Evaluate $\int_0^{\frac{\pi}{2}} (3\sin x\hat{i} + 2\cos x\hat{j})dx$. 4

8. (a) Find the potential function f for the field $\vec{F} = x\hat{i} + y^2\hat{j} + 4z\hat{k}$. 5

(b) Evaluate $\int_C (y^2x + y)dx + (x^2y + x)dy$

where C is the line segment from (1,1) to (2,3). 5

(c) Find the flux of the field $\vec{F} = 2x\hat{i} - 3y\hat{j}$ outward across the ellipse $x = \cos t, y = 4\sin t; 0 \le t \le 2\pi$. 4

UNIT–V

9. (a) Use Green's theorem to find the area enclosed by the ellipse $x = a\cos\theta, y = b\sin\theta, 0 \le \theta \le 2\pi$. 5

(b) Verify Green's theorem in the plane $\int_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$

where C is the boundary of the region defined by $y = \sqrt{x}, y = x^2$. 5

(c) Find the area of the portion cut from the paraboloid $x^2 + y^2 - z = 0$ by the planes z=0 and z=10. 4

10. (a) Verify Stroke's theorem for $A = (2x - y)\hat{i} - yz^2\hat{j} - y^2 z\hat{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. 5

(b) Use Green's theorem, evaluate $\int_C \left(y^2 dx + x^2 dy\right)$

where C is the triangle bounded by $z = 0, x + y = 1, y = 0$. 5

(c) Verify divergence theorem for the sphere $x^2 + y^2 + z^2 = a^2$ if $\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}$. 4
