2022

B.A./B.Sc.

Third Semester

CORE – 7

MATHEMATICS

Course Code: MAC 3.31

(PDE & Systems of ODE)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) Form the PDE by eliminating constants r and c from $x^2 + y^2 + (z-c)^2 = r^2$. 3

   (b) Find the general solution of the PDE $(y+ux)u_x - (x+yu)u_y = x^2 - y^2$. 5

   (c) Obtain the solution of the equation $(y-u)u_x + (u-x)u_y = x-y$ with the condition $u = 0$ on $xy = 1$. 6

2. (a) Form the PDE arising from the surface $u = x + y + f(xy)$. 3

   (b) Use the method of separation of variables to solve $u_x = 2u_y + u$, $u(x,0) = 6e^{-3x}$. 5

   (c) Reduce the first order PDE $yu_x + u_y = x$ to canonical form and obtain its general solution. 6

UNIT–II

3. (a) Classify the second order PDE $u_{xx} - 2u_{xy} - 8u_{yy} = 0$ and find its characteristic curves. 3

   (b) Reduce the second order PDE $x^2u_{xx} - y^2u_{yy} = 0$ to canonical form. 5

   (c) Derive the 2D Laplace's equation. 6

4. (a) Determine the region in the *xy*-plane where the second order PDE

$u_{xx} + yu_{yy} + \frac{1}{2}u_y = 0$ is hyperbolic. 3

(b) Derive the one-dimensional heat equation. 5

(c) Find the solution of the second order PDE

$x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = 0$ by reducing it to canonical form. 6

UNIT–III

5. (a) Determine the solution of the initial value problem: 4

$u_{tt} = u_{xx},\ 0 < x < \infty,\ t > 0$

$u(x,0) = \cos\left(\frac{\pi x}{2}\right),\ 0 \le x < \infty$

$u_t(x,0) = 0,\ 0 \le x < \infty$

$u_x(x,0) = 0,\ t \ge 0$

(b) Determine the solution of the initial value problem: 4

$u_{tt} - c^2u_{xx} = e^x, u(x,0) = 5, u_t(x,0) = x^2$

(c) Solve the heat conduction equation

$u_t = ku_{xx},\ 0 < x < l,\ t > 0$

$u(0,t) = 0,\ u(l,t) = 0,\ t \ge 0$

$u(x,0) = f(x),\ 0 \le x \le l$

by the method of separation of variables. 6

6. (a) Let $u(x,t)$ be the generalised solution of: 2+2=4

$u_{tt} - u_{xx} = 0,\ t > 0$

$$u(x,0) = \begin{cases} x(1-x) & \text{if } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

$u_t(x,0) = 0$

Evaluate:

(i) $u\left(\frac{-1}{2}, \frac{3}{4}\right)$ (ii) $u\left(\frac{1}{2}, \frac{3}{4}\right)$

(b) Determine the solution of the initial value problem:

$u_{tt} - c^2 u_{xx} = x,\ u(x,0) = 0,\ u_t(x,0) = 3$ 4

(c) Obtain solutions of 6

$$u_{tt} = c^2 u_{xx} = x,\ x > 0, t > 0$$
$$u(x,0) = f(x),\ x \geq 0$$
$$u_t(x,0) = g(x),\ x \geq 0$$
$$u(0,t) = p(t),\ t \geq 0$$

UNIT–IV

7. (a) Apply operator method to find the general solution of the linear system: $\left.\begin{array}{l} x'' + y' = e^{2t} \\ x' + y' - x - y = 0 \end{array}\right\}$ 8

(b) Consider the linear system $\left.\begin{array}{l} x' = 5x + 3y \\ y' = 4x + y \end{array}\right\}$ 2+1+3=6

(i) Show that $\left.\begin{array}{l} x = 3e^{7t} \\ y = 2e^{7t} \end{array}\right\}$ and $\left.\begin{array}{l} x = e^{-t} \\ y = -2e^{-t} \end{array}\right\}$ are linearly independent solutions $\forall t \in [a,b]$.

(ii) Write the general solution.

(iii) Find the solutions such that $x = f(t), y = g(t)$ such that $f(t) = 0$, $g(0) = 8$.

8. (a) Convert $x^{iv} + 7x''' + 4x'' + 5x' - 2x = 0$ into a system of first order differential equations. 4

(b) Apply the transformation $t = e^w$ to convert the system $\left.\begin{array}{l} tx' = x + y \\ ty' = -3x + 5y \end{array}\right\}$ into a system with constant coefficient and hence find the general solution. 4

(c) Obtain the general solution of the linear system 6

$$\left.\begin{array}{l} x' = 7x + 4y \\ y' = -x + 3y \end{array}\right\}$$

UNIT–V

9. (a) Consider the initial value problem: $y' = x - 2y, y(0) = 1$
Apply the Euler's method to approximate the value of the solution y at $x = 0.2$ and 0.4 taking $h = 0.2$. 4

(b) Consider the initial value problem: $y' = x + y, y(0) = 1$, apply the fourth order Runge-Kutta method to approximate the value (up to the 5th decimal places) of the solution y at $x = 0.1$ with $h = 0.1$. 4

(c) Use the method of successive approximations to find the first three members $\varphi_1, \varphi_2, \varphi_3$ of a sequence of functions that approaches the exact solution of the initial value problem $y' = e^x + y^2, y(0) = 0$. 6

10. (a) Use the method of successive approximations to find the first three members $\varphi_1, \varphi_2, \varphi_3$ of a sequence of functions that approaches the exact solution of the initial value problem $y' = xy, y(0) = 1$. 4

(b) Discuss Range-Kutta method for approximating the values of the solution of the initial value problem $y' = f(x, y), y(x_0) = y_0$. 4

(c) By using an appropriate example, show that the improved Euler's method is a better approximation than the Euler's method. 6