2022
B.A./B.Sc.
First Semester
CORE – 1
MATHEMATICS
Course Code: MAC 1.11
(Calculus)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) Prove that $\tanh^{-1} x = \frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$. 4

(b) Find the n^{th} derivatives of $y = \frac{x^2}{(x+2)(2x+3)}$ by partial fraction method. 5

(c) If $y = \cos\left(m\sin^{-1} x\right)$, show that
$\left(1-x^2\right)y_{n+2} - (2n+1)xy_{n+1} + \left(m^2 - n^2\right)y_n = 0$
and hence find $y_n(0)$. 3+2=5

2. (a) State and prove Leibnitz's theorem. 1+5=6

(b) Consider the function $f(x) = xe^{-x}$. Find the interval in which f decreases and increases, concave up and concave down. Also, locate the point of inflection. 2+2+1=5

(c) Find the asymptote of the hyperbolic spiral $r\theta = a$ 3

UNIT–II

3. (a) An open-top box is to be made by cutting small congruent squares from the corner of a (12 × 12) inch sheet of tin and bending up the sides. How large should the squares cut from the corners be to make the box hold as much as possible? 5

(b) Sketch the graph of a function $f(x)=\frac{x^2+4}{2x}$ and label the coordinate of the stationary points. 5

(c) Find the values of a and b such that

$$\lim_{x\to 0}\frac{x(1-a\cos x)+b\sin x}{x^3}=\frac{1}{3}$$

assuming L'Hospital's rule is applicable. 4

4. (a) Trace the curve $r=a(1+\cos\theta), a>0$. 5

(b) It is given that the cost function $c(x)=5000+15x+0.01x^2$

(i) Find the total cost, average cost and marginal cost to produce 100 items. 2

(ii) Find the production level that will minimize the average cost. 2

(iii) Find the minimum average cost. 1

(c) Find $\lim_{x\to 0}(\sin x)^{2\tan x}$ 4

UNIT–III

5. (a) Derive the reduction formula for $\int \sin^m x\cos^n x\,dx$ and hence evaluate $\int \sin^4 x\cos^3 x\,dx$. 5

(b) Find the volume of the solid generated by revolving the region bounded by $y=\sqrt{x}$ and the lines $y=1, x=4$ about the line $y=1$. 5

(c) Find the length of the curve which is given by the graph of the function $y=\frac{4\sqrt{2}}{3}x^{\frac{3}{2}}-1,\ 0\le x\le 1$. 4

6. (a) Use cylindrical shell to find the volume of the solid generated when the region enclosed between $y=\sqrt{x}, x=1, x=4$ and the x-axis is revolved about the y-axis. 5

(b) Show that the area of the surface of a sphere of radius r is $4\pi r^2$. 5

(c) Find the arc length of the curve $x = e^t \cos t$, $y = e^t \sin t$ in the interval $0 \le t \le \frac{\pi}{2}$. 4

UNIT–IV

7. (a) Identify and sketch the graph of the equation $16x^2 + 9y^2 - 64x - 54y + 1 = 0$. 5

(b) Rotate the coordinate axes to remove the *xy*-term in the equation $x^2 - xy + y^2 - 2 = 0$. Also, identify its graph. 5

(c) Find a polar equation for the conic with eccentricity $\frac{4}{3}$ and directrix $y = 3$. 4

8. (a) Sketch the graph of the ellipse $25x^2 - 14xy + 25y^2 - 288 = 0$ and label its foci, vertices and the ends of the minor axis. 5

(b) Let $x'y'$-coordinate system be obtained by rotating *xy*-coordinate system through an angle 45°. Find an equation of the curve $3x'^2 + y'^2 = 6$ in the *xy*-coordinates. 5

(c) Find the eccentricity and directrix of the parabola

$$r = \frac{3}{2 - 2\cos\theta}$$ 4

UNIT–V

9. (a) Evaluate the definite integral $\int_0^2 \left\| t\hat{i} + t^2 \hat{j} \right\| dt$. 4

(b) State and prove Kepler's second law. 1+3=4

(c) A shell is fired from ground level with a muzzle speed 320 ft/s and elevation angle of 60°. Find the following: 2×3=6

(i) parametric equations for the shell's trajectory

(ii) maximum height reached by the shell

(iii) horizontal distance travelled by the shell

10. (a) A shell is to be fired from the ground level at an elevation angle of 30°. What should the muzzle speed be in order for the maximum height of the shell to be 2500 ft? 5

(b) Let $\vec{r}(t) = \cos t\,\hat{i} + \sin t\,\hat{j} + \hat{k}$. Find $\lim_{t\to 0}\left(\vec{r}(t)\times\vec{r'}(t)\right)$. 4

(c) The position function of a particle is given as $\vec{r} = e^{-1}\hat{i} + e^{t}\,\hat{j}$.
Find the scalar tangential and normal components of the acceleration at time $t = 0$. 2½+2½=5
