May 2025
M.Sc.
Fourth Semester
DISCIPLINE SPECIFIC ELECTIVE – 04
MATHEMATICS
Course Code: MMAD 4.21
(Fluid Mechanics)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five *questions, taking* one *from each unit.*

UNIT–I

1. (a) Define material, local, and convective derivatives. Find the stream lines and path lines of the particles for the velocity field $u = \frac{x}{1+t}, v = y, w = 0$. 3+4=7

   (b) Show that the surface $\frac{x^2}{a^2k^2t^4} + kt^2\left[\left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2\right] = 1$ is a possible form of boundary surface of a liquid at time *t*. 7

2. (a) Derive the Euler equations of motion of an ideal fluid. 7
   (b) Derive the equation of energy of an inviscid fluid flow. 7

UNIT–II

3. (a) Find the image of a simple source with respect to a line and show that the image of a doublet with respect to a line is an equal doublet symmetrically placed. Also, find the complex potential due to this doublet system. 3+2+2=7
   (b) State and prove the theorem of Blasius. 7

4 (a) What arrangement of sources and sinks will give rise to the complex potential function $w = \ln\left(z - \frac{a^2}{z}\right)$? 7

   (b) State and prove the Milne-Thomson circle theorem. 7

UNIT–III

5. (a) Prove that the product of the cross section and vorticity at any point on a vortex filament is constant along the filament. 7
   (b) Determine the stream function when the strengths of the vortex filaments are equal. 7

6 (a) Find the complex potential w and the stream function ψ due to a set of line vortices of strength k placed at points $z = \pm na \ (n = 0,1,2,3...)$ 7
   (b) Describe Karman vortex street and find velocity components at the origin. 7

UNIT–IV

7 (a) Derive the Navier-Stokes equations of motion for viscous fluid. 7
   (b) Discuss the Hagen-Poiseuille flow. 7

8 (a) Show that the general motion of a fluid element can be expressed as the combination of translation, rotation, and deformation of the fluid element. 7
   (b) Derive diffusion of vorticity equation. 7

UNIT–V

9. (a) Derive the equation of motion of a gas in isentropic flow. 7
   (b) Derive an expression for the speed of sound in gas. An aeroplane is flying at a height of 14 km where the temperature is –45°C. The speed of the plane corresponds to Mach number 2. Find the speed of the plane. Take R = 287 J/kg K and γ =1.4. 5+2=7

10 (a) Derive energy equation for compressible flow. 7
   (b) A 120 mm diameter pipe reduced to 60 mm diameter through a sudden contraction. When it carries air at 25°C under isothermal condition, the absolute pressure observed in the two pipes just before and after contraction are 480 K N/m^2 and 384 K N/m^2 respectively. Determine 3½×2=7
   (i) velocities at the two sections
   (ii) mass rate of flow through the pipe
   Take R = 287 J/kg K.